§ 2.6: Modeling with Functions

Modeling with Functions

Example 1	Modeling the Volume of a Box

A breakfast cereal company manufactures boxes to package their product. For aesthetic reasons, the box must have the following proportions: Its width is 3 times its depth and its height is 5 times its depth.

(a) Find a function that models the volume of the box in terms of its depth.

(b) Find the volume of the box if the depth in 1.5 in.

(c) For what depth is the volume 90 in^3?

(d) For what depth is the volume greater than 60 in^3?

To find the function that models the volume of the box, we use the following steps.

- **Express the Model in Words**
  We know that the volume of a rectangular box is
  $$\text{volume} = \text{depth} \times \text{height} \times \text{width}$$
- **Choose the Variable**
  There are three varying quantities – width, depth, and height. Since the function we want depends on the depth, we let
  $$x = \text{depth of the box}$$
  Then we express the other dimensions of the box in terms of x.

In words	In Algebra
Depth	x
Width	$3x$
Height	$5x$

- **Set up the Model**
  The model is the function V that gives the volume of the box in terms of the depth x.
  $$\begin{aligned} \text{volume} &= \text{depth} \times \text{width} \times \text{height} \\ V(x) &= x \times 3x \times 5x \\ V(x) &= 15x^3 \end{aligned}$$

➢ **Use the Model**

We use the model to answer the questions in parts (b), (c), and (d).

(b) If the depth is 1.5 in., the volume is

$$V(1.5) = 15(1.5)^3 = 50.625\ in^3.$$

(c) We need to solve the equation $V(x) = 90$ or

$$\begin{aligned} 15x^3 &= 90 \\ x^3 &= 6 \\ x &= \sqrt[3]{6} \approx 1.82\ in \end{aligned}$$

(d) We need to solve the equation $V(x) > 60$ or

$$\begin{aligned} 15x^3 &> 60 \\ x^3 &> 4 \\ x &> \sqrt[3]{4} \approx 1.59\ in \end{aligned}$$

Guidelines for Modeling with Functions

1. Express the Model in Words. Identify the quantity you want to model and express it, in words, as a function of the other quantities in the problem.

2. Choose the Variable. Identify all the variables used to express the function in step 1. Assign a symbol, such as x, to one variable and express the other variables in terms of this symbol.

3. Set up the Model. Express the function in the language of algebra by writing it as a function of the single variable chosen in Step 2.

4. Use the Model. Use the function to answer the questions posed in the problem.

Example 2	Fencing a Garden

A gardener has 140 feet of fencing to fence in a rectangular vegetable garden.

(a) Find a function that models the area of the garden she can fence.

(b) For what range of widths is the area greater than or equal to 825 ft^2?

(c) Can she fence a garden with area 1250 ft^2?

(d) Find the dimensions of the largest area she can fence.

Example 3	Length

A woman 5 ft tall is standing near a street lamp that is 12 ft tall. Find a function that models the length L of her shadow in terms of her distance d from the base of the lamp.

Example 4	Area

A rectangle is inscribed in a semicircle of radius 10. Find a function that models the area A of the rectangle in terms of its height h.

Example 5	Stadium Revenue

A baseball team plays in a stadium that holds 55,000 spectators. With the ticket price at $10, the average attendance at recent games has been 27,000. A market survey indicates that for every dollar the ticket price is lowered, attendance increases by 3000.

(a) Find a function that models the revenue in terms of ticket price.

(b) What ticket price is so high that no revenue is generated?

(c) Find the price that maximizes revenue from ticket price.

Example 6	Bird Flight

A bird is released from a point on an island 5 mi from the nearest point on a straight shoreline. The bird flies to a point on the shoreline, and then flies along the shoreline to its nesting area. Suppose the bird requires 10 kcal/mi of energy to fly over land and 14 kcal/mi to fly over water.

(a) Find a function that models the energy expenditure of the bird.

(b) If the bird instinctively chooses the path that minimizes its energy expenditure, to what point does it fly?

Homework

Due:_____

2 – 30 even (except 12)