HW #3 Solutions

Problem # 3.3

a) Let $E$ be the event that exactly three of the four tosses come up heads. Then

$$P(E) = P(HHHT) + P(HHTH) + P(HTHH) + P(THHH)$$

$$= 4P(HHHT) = 4 \times \frac{1}{2^4} = \frac{1}{4}$$

b) Let $E$ be the event that the last two tosses come up heads. Then

$$P(E) = P(HHHH) + P(HTHH) + P(THHH) + P(TTHH)$$

$$= 4P(HHHH) = 4 \times \frac{1}{2^4} = \frac{1}{4}$$

c) Let $E$ be the event that the third toss comes up heads. By direct enumeration of the outcomes, we know that there are eight possible outcomes each of which has the same probability as $P(HTHT)$. Hence

$$P(E) = 8P(HTHT) = 8 \times \frac{1}{2^4} = \frac{1}{2}$$

d) Let $E$ be the event that all tosses come up the same. Then

$$P(E) = P(HHHH) + P(TTTT) = 2P(HHHH) = 2 \times \frac{1}{2^4} = \frac{1}{8}$$

Problem # 3.6

To sketch a Venn diagram and label the probabilities of the regions, we need to compute the following probabilities:

$$P(E \cup F) = P(E) + P(F) - P(EF) = 0.40 + 0.55 - 0.15 = 0.8.$$  

$$P(E^c) = P(E) - P(EF) = 0.40 - 0.15 = 0.25.$$  

$$P(F^c) = P(F) - P(EF) = 0.55 - 0.15 = 0.4.$$
\[ P((E \cup F)^c) = 1 - P(E \cup F) = 1 - 0.8 = 0.2. \]

Problem # 3.8
a.
\[ P(EF) = P(E)P(F|E) = 0.55 \times 0.20 = 0.11 \]

b.
\[ P(E^c \cup F^c) = 1 - P(EF) = 1 - 0.11 = 0.89. \]
c.
\[
\begin{align*}
P(E^c F^c) &= P((E \cup F)^c) = 1 - P(E \cup F) = 1 - (P(E) + P(F) - P(EF)) \\
&= 1 - (0.55 + 0.40 - 0.11) = 0.16.
\end{align*}
\]
d.
\[
P(E|F) = \frac{P(EF)}{P(F)} = \frac{0.11}{0.40} = 0.275.
\]

Problem # 3.9
a.
\[ P(EF) = P(E)P(F|E) = 0.50 \times 0.30 = 0.15. \]

b.
\[ P(E \cup F) = P(E) + P(F) - P(EF) = 0.50 + 0.20 - 0.15 = 0.55. \]
c.
\[ P(E^c F^c) = 1 - P(E \cup F) = 1 - 0.55 = 0.45. \]
d.
\[
P(E|F) = \frac{P(EF)}{P(F)} = \frac{0.15}{0.20} = 0.75.
\]

Problem #3.10
Let E and F be the event that the husband, respectively, wife will watch the 2003 Superbowl football game.

a. 

\[ P(EF) = P(E)P(F|E) = 0.20 \times 0.25 = 0.05. \]

b. 

\[ P(E \cup F) = P(E) + P(F) - P(EF) = 0.20 + 0.08 - 0.05 = 0.23. \]

c. 

\[ P(E^cF^c) = 1 - P(E \cup F) = 1 - 0.23 = 0.77. \]

d. 

\[ P(E|F) = \frac{P(EF)}{P(F)} = \frac{0.05}{0.08} = 0.625. \]

Problem #3.11

Let E be the event that at least one ace. Then

\[ P(E) = 1 - P(E^c) = 1 - \left(\frac{5}{6}\right)^4 = \frac{671}{1296} = 0.5177 \]

The addition rule cannot be used here because these events are not mutually exclusive.