§ 2.1

2 – 8 (even), 14 – 28 (even), 32, 34, 36, 44, 50, 52, 58, 62, 66, 70

Solutions

1 – 4 Express the rule in function notation

2) Divide by 7, then subtract 4

\[ f(x) = \frac{x}{7} - 4 \]

4) Take the square root, add 8, then multiply by \( \frac{1}{3} \)

\[ f(x) = \frac{1}{3}(\sqrt{x} + 8) \]

5 – 8 Express the function in words

6) \( g(x) = \frac{x}{3} - 4 \)

Divide by 3, then subtract 4

8) \( k(x) = \sqrt{x + 2} \)

Add 2, then take the square root

13 – 20 Evaluate the function at the indicated values

14) \( f(x) = x^2 + 2x \)

\[ f(0) = 0^2 + 2(0) = 0 \]

\[ f(3) = 3^2 + 2(3) = 15 \]

\[ f(a) = a^2 + 2a \]

\[ f(-x) = (-x)^2 + 2(-x) = x^2 - 2x \]

\[ f\left(\frac{1}{a}\right) = \left(\frac{1}{a}\right)^2 + 2\left(\frac{1}{a}\right) = \frac{1}{a^2} + \frac{2}{a} = \frac{1 + 2a}{a^2} \]
16) \( h(t) = t + \frac{1}{t} \)

\[
h(1) = 1 + \frac{1}{1} = 2
\]

\[
h(-1) = -1 + \frac{1}{-1} = -1 - 1 = -2
\]

\[
h(2) = 2 + \frac{1}{2} = \frac{5}{2}
\]

\[
h\left(\frac{1}{2}\right) = \frac{1}{2} + \frac{1}{\frac{1}{2}} = \frac{1}{2} + 2 = \frac{5}{2}
\]

\[
h(x) = x + \frac{1}{x}
\]

\[
h\left(\frac{1}{x}\right) = \frac{1}{x} + \frac{1}{\frac{1}{x}} = \frac{1}{x} + x = h(x)
\]

18) \( f(x) = x^3 - 4x^2 \)

\[
f(0) = 0^3 - 4(0)^2 = 0
\]

\[
f(1) = 1^3 - 4(1)^2 = -3
\]

\[
f(-1) = (-1)^3 - 4(-1)^2 = -1 - 4 = -5
\]

\[
f\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^3 - 4\left(\frac{3}{2}\right)^2 = \frac{27}{8} - 9 = \frac{27}{8} - \frac{72}{8} = -\frac{45}{8}
\]

\[
f\left(\frac{x}{2}\right) = \left(\frac{x}{2}\right)^3 - 4\left(\frac{x}{2}\right)^2 = \frac{x^3}{8} - x^2 = \frac{x^3}{8} - \frac{8x^2}{8} = \frac{x^2(x - 8)}{8}
\]

\[
f(x^2) = (x^2)^3 - 4(x^2)^2 = x^6 - 4x^4 = x^4(x^2 - 4) = x^4(x + 2)(x - 2)
\]
20) \( f(x) = \frac{|x|}{x} \)

\[ f(-2) = \frac{|-2|}{-2} = \frac{2}{-2} = -1 \]

\[ f(-1) = \frac{|-1|}{-1} = \frac{1}{-1} = -1 \]

\( f(0) \) is undefined

\[ f(5) = \frac{|5|}{5} = \frac{5}{5} = 1 \]

\[ f(x^2) = \frac{|x^2|}{x^2} = \frac{x^2}{x^2} = 1 \]

\[ f\left(\frac{1}{2}\right) = \left|\frac{1}{x}\right| = \frac{1}{|x|} = x \]

21 – 24 Evaluate the piecewise defined function at the indicated values

22)

\[ f(x) = \begin{cases} 5 & x \leq 2 \\ 2x - 3 & x > 2 \end{cases} \]

\[ f(-3) = 5 \]

\[ f(0) = 5 \]

\[ f(2) = 5 \]

\[ f(3) = 2(3) - 3 = 3 \]

\[ f(5) = 2(5) - 3 = 7 \]
24) 

\[ f(x) = \begin{cases} 
3x & x < 0 \\
(x + 1) & 0 \leq x \leq 2 \\
(x - 2)^2 & x > 2 
\end{cases} \]

\[ f(-5) = 3(-5) = -15 \]

\[ f(0) = 0 + 1 = 1 \]

\[ f(1) = 1 + 1 = 2 \]

\[ f(2) = 2 + 1 = 3 \]

\[ f(5) = (5 - 2)^2 = 3^2 = 9 \]

25 – 28 Use the function to evaluate the indicated expressions and simplify

26) \( f(x) = 3x - 1 \)

\[ f(2x) = 3(2x) - 1 = 6x - 1 \]

\[ 2f(x) = 2(3x - 1) = 6x - 2 \]

28) \( f(x) = 6x - 18 \)

\[ f\left(\frac{x}{3}\right) = 6\left(\frac{x}{3}\right) - 18 = 2x - 18 \]

\[ \frac{f(x)}{3} = \frac{6x - 18}{3} = 2x - 6 \]

29 – 36 Find \( f(a) \), \( f(a + h) \), and the difference quotient \( \frac{f(a+h) - f(a)}{h} \), where \( h \neq 0 \).

32) \( f(x) = x^2 + 1 \)

\[ f(a) = a^2 + 1 \]

\[ f(a + h) = (a + h)^2 + 1 = a^2 + 2ah + h^2 + 1 \]

\[ \frac{f(a+h) - f(a)}{h} = \frac{a^2 + 2ah + h^2 + 1 - a^2 - 1}{h} = \frac{2ah + h^2}{h} = 2a + h \]
34) \( f(x) = \frac{2x}{x-1} \)

\[ f(a) = \frac{2a}{a - 1} \]

\[ f(a + h) = \frac{2(a + h)}{a + h - 1} = \frac{2a + 2h}{a + h - 1} \]

\[ \frac{f(a + h) - f(a)}{h} = \frac{\frac{2a + 2h}{a + h - 1} - \frac{2a}{a - 1}}{h} = \frac{-2ah}{(a + h - 1)(a - 1)} \]

36) \( f(x) = x^3 \)

\[ f(a) = a^3 \]

\[ f(a + h) = (a + h)^3 = a^3 + 3a^2h + 3ah^2 + h^3 \]

\[ \frac{f(a + h) - f(a)}{h} = \frac{a^3 + 3a^2h + 3ah^2 + h^3 - a^3}{h} = \frac{3a^2h + 3ah^2 + h^3}{h} \]

\[ = 3a^2 + 3ah + h^2 \]

37 - 58 Find the domain of the function

44) \( f(x) = \frac{x^4}{x^2+x-6} = \frac{x^4}{(x+3)(x-2)} \)

Domain of \( f = \{ x : x \neq -3 \text{ and } x \neq 2 \} \)

50) \( G(x) = \sqrt{x^2 - 9} \)

\[ x^2 - 9 \geq 0 \]

\[ x^2 \geq 9 \]

\[ x \leq -3 \text{ or } x \geq 3 \]

Domain of \( G = \{ x : x \leq -3 \text{ or } x \geq 3 \} \)
52) \( g(x) = \frac{\sqrt{x}}{2x^2 + x - 1} = \frac{\sqrt{x}}{(2x - 1)(x + 1)} \)

Domain of \( g = \{x \geq 0 : x \neq \frac{1}{2}\} \)

58) \( f(x) = \frac{x}{\sqrt{9 - x^2}} \)

\[ 9 - x^2 \geq 0 \]
\[ x^2 \leq 9 \]
\[ -3 \leq x \leq 3 \]

Domain of \( f = \{x : -3 \leq x \leq 3\} \)
62) **Toricelli’s Law**

A tank holds 50 gallons of water, which drains from a leak at the bottom, causing the tank to empty in 20 minutes. The tank drains faster when it is nearly full because the pressure on the leak is greater. **Toricelli’s Law** gives the volume of water remaining in the tank after \( t \) minutes as

\[
V(t) = 50 \left(1 - \frac{t}{20}\right)^2 \quad 0 \leq t \leq 20
\]

(a) Find \( V(0) \) and \( V(20) \)

\[
V(0) = 50 \left(1 - \frac{0}{20}\right)^2 = 50 \text{ gallons}
\]

\[
V(20) = 50 \left(1 - \frac{20}{20}\right)^2 = 0 \text{ gallons}
\]

(b) What do your answers from part (a) represent?

The initial and final volumes of the tank.

(c) Make a table of values of \( V(t) \) for \( t = 0,5,10,15,20 \).

<table>
<thead>
<tr>
<th>( t )</th>
<th>( V(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>5</td>
<td>( \frac{225}{8} )</td>
</tr>
<tr>
<td>10</td>
<td>( \frac{25}{2} )</td>
</tr>
<tr>
<td>15</td>
<td>( \frac{25}{8} )</td>
</tr>
<tr>
<td>20</td>
<td>0</td>
</tr>
</tbody>
</table>
66) **Income Tax**

In a certain country, income tax $T$ is assessed according to the following function of income $x$:

\[ T(x) = \begin{cases} 
0 & 0 \leq x \leq 10,000 \\
0.08x & 10,000 < x \leq 20,000 \\
1600 + 0.15x & 20,000 < x
\end{cases} \]

(a) Find $f(5000)$, $f(12,000)$, and $f(25,000)$

$f(5000) = 0$

$f(12,000) = 0.08(12,000) = 960$

$f(20,000) = 1600 + 0.15(25,000) = 5,350$

(b) What do your answers from part (a) represent?

The amount of tax owed for $5,000, $12,000, and $25,000 incomes respectfully.

70) **Height of Grass**

A home owner mows a lawn every Wednesday afternoon. Sketch a rough graph of the height of the grass as a function of time over the course of a four week period beginning on a Sunday.