§ 2.5: Quadratic Functions; maxima and Minima

Graphing Quadratic Functions Using the Standard Form

A quadratic function is a function \( f \) of the form

\[
f(x) = ax^2 + bx + c
\]

where \( a, b, \) and \( c \) are real numbers and \( a \neq 0 \).

For example, if we take \( a = 1, b = c = 0 \), we get the simple quadratic function

\[
f(x) = x^2.
\]

The graph of any quadratic function is called a parabola, and can be obtained from the graph of \( f(x) = x^2 \) by the transformations discussed in § 2.4.

**Standard Form of a Quadratic Function**

A quadratic function \( f(x) = ax^2 + bx + c \) can be expressed in the standard form

\[
f(x) = a(x - h)^2 + k
\]

by completing the square. The graph of \( f \) is a parabola with vertex \( (h,k) \); the parabola opens upward if \( a > 0 \) or downward if \( a < 0 \).

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<th>Example 1</th>
<th>Standard Form of a Quadratic Function</th>
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<td>Let ( f(x) = 3x^2 - 12x + 20 ).</td>
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<tr>
<td>(a) Express ( f ) in standard form.</td>
<td></td>
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<tr>
<td>(b) Sketch the graph of ( f ).</td>
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</table>
Maximum and Minimum Values of Quadratic Functions

**Maximum or Minimum Value of a Quadratic Function**

Let \( f \) be a quadratic function with standard form \( f(x) = a(x - h)^2 + k \). The maximum or minimum value of \( f \) occurs at \( x = h \).

If \( a > 0 \), then the **minimum value** of \( f \) is \( f(h) = k \).

If \( a < 0 \), then the **maximum value** of \( f \) is \( f(h) = k \).

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**Example 2**  | **Minimum Value of a Quadratic Function**

Consider the quadratic function \( f(x) = 5x^2 + 20x + 19 \).

(a) Express \( f \) in standard form.

(b) Find the minimum value of \( f \).
Example 3  
Maximum Value of a Quadratic Function

Consider the quadratic function \( f(x) = -2x^2 + 14x - 10 \).

(a) express \( f \) in standard form.

(b) Find the maximum value of \( f \).

We now derive a formula for the maximum or minimum of the quadratic function \( f(x) = ax^2 + bx + c \) in terms of \( a, b, \) and \( c \).

\[
f(x) = ax^2 + bx + c \]

\[
= a \left( x^2 + \frac{b}{a}x \right)^2 + c \quad \text{(Factor } a \text{ from the } x\text{-terms)}
\]

\[
= a \left( x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} \right) + c - a \left( \frac{b^2}{4a^2} \right) \quad \text{(Complete the square)}
\]

\[
= a \left( x + \frac{b}{2a} \right)^2 + c - \frac{b^2}{4a} \quad \text{(Factor and simplify)}
\]
Thus we get

**Maximum of Minimum Value of a Quadratic Function**

The maximum or minimum value of a quadratic function $f(x) = ax^2 + bx + c$ occurs at

$$x = -\frac{b}{2a},$$

the maximum or minimum value is

$$f \left( -\frac{b}{2a} \right) = c - \frac{b^2}{4a}$$

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**Example 4** | Finding Maximum and Minimum Values of Quadratic Functions
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Find the maximum or minimum value of each quadratic function.

(a) $f(x) = x^2 + 4x$  
(b) $f(x) = -2x^2 + 4x - 5$
Example 5  Advertising

The effectiveness of a television commercial depends on how many times a viewer watches it. After some experiments and advertising agency found that if the effectiveness $E$ is measured on a scale of 1 to 10, then

$$E(n) = \frac{2}{3} n - \frac{1}{90} n^2$$

where $n$ is the number of times a viewer watches a given commercial. For a commercial to have maximum effectiveness, how many times should a viewer watch it?
Example 6  

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<th>Domain and Range of Quadratic Functions</th>
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Find the domain and range of each of the following quadratic functions.

(a) \( f(x) = -x^2 + 4x - 3 \)  
(b) \( f(x) = 2x^2 + 6x - 7 \)

Homework

Due: ____________________________

6 – 44 (even)