8.8 Applications of Power Series

- The Maclaurin series generated by \( f(x) = (1 + x)^m \), when \( m \) is constant, is

\[
1 + mx + \frac{m(m-1)}{2!}x^2 + \frac{m(m-1)(m-2)}{3!}x^3 + \ldots + \frac{m(m-1)(m-2)\ldots(m-k+1)}{k!}x^k + \ldots
\]

This series, called the binomial series, converges absolutely for \( |x| < 1 \). It can also be shown that for \( -1 < x < 1 \),

\[
(1 + x)^m = 1 + \sum_{k=1}^{\infty} \binom{m}{k} x^k,
\]

where we define

\[
\binom{m}{1} = m, \quad \binom{m}{2} = \frac{m(m-1)}{2!},
\]

and

\[
\binom{m}{k} = \frac{m(m-1)(m-2)\ldots(m-k+1)}{k!} \quad \text{for } k \geq 3.
\]

[Example 1 and 2]

- **Series Solutions of Differential Equations.** When we cannot find a relatively simple expression for the solution of an initial value problem or differential equation, we may try to find a power series representation for the solution. If we can do so, we immediately have a source of polynomial approximations of the solution, which may be all that we really need. [20,30]

- We can sometimes evaluate indeterminate forms by expressing the functions involved as Taylor series.[38, Example 6]