1. (15 pts.) \( y = e^{-2x} - 2 \)
   
   (a) (4 pts.) \( \frac{dy}{dx} = -2e^{-2x} \)
   
   (b) (11 pts.) When \( x = 0 \), \( \frac{dy}{dx} = -2 \). The point-slope equation of the line with slope \(-2\) and a point \((0, -1)\) is \( y + 1 = -2x \). Therefore the tangent line to the graph of the function at the point \((0, -1)\) is \( y = -2x - 1 \).

2. (15 pts.) \( \frac{d}{dx} (x^2y - 2x^3 - y^3 + 677) = \frac{d}{dx} 0 \Rightarrow x^2 \frac{dy}{dx} + 2xy - 6x^2 - 3y^2 \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{6x^2 - 6x^2}{x^2 - 3y^2} \).
   
   At \((-2, 9)\), \( \frac{dy}{dx} = \frac{24 - 2(-2)(0)}{4 - 3(81)} \). Therefore the slope of the curve at \((-2, 9)\) is \( -\frac{60}{239} \).

3. (40 pts.)
   
   (a) (20 pts.) The \( y \)-intercept is 7.
   
   \[
   f'(x) = 6x^2 + 42x - 48 = 6 \left( x^2 + 7x - 8 \right) = 6 \left( x + 8 \right) \left( x - 1 \right) \cdot
   
   f''(x) = 12x + 42 = 6(2x + 7) \). The domain of \( f \) is \(( -\infty, \infty )\). The critical points are \(-8\) and 1. Since \( f''(-8) < 0 \) and \( f(-8) = 711 \), \((-8, 711)\) is a relative maximum. Since \( f''(1) > 0 \) and \( f(1) = -18 \), \((1, -18)\) is a relative minimum. Since \( f'' \left( -\frac{7}{2} \right) = 0 \), \(-\frac{7}{2} \) may be an inflection point. Using test value \(-4 \) and 0, we find that \( f''(-4) < 0 \) and \( f''(0) > 0 \), so \((-3.5, 346.5)\) is an inflection point. The graph of \( f \) is sketched below.

   (b) (20 pts.) The \( y \)-intercept is \(-\sqrt[4]{4} \approx -1.5874 \). The \( x \)-intercept is \( \frac{2}{3} \).
   
   \[
   f'(x) = \frac{1}{3} (6x - 4)^{-2/3} (6) = \frac{2}{(6x - 4)^{2/3}} \cdot
   
   f''(x) = -\frac{4}{3} (6x - 4)^{-5/3} (6) = -\frac{8}{(6x - 4)^{5/3}} \). The domain of \( f \) is \(( -\infty, \infty )\). The critical point is \( \frac{2}{3} \). Using test value 0 and 1, we find that \( f'(0) > 0 \) and \( f'(1) > 0 \), so there are no relative extrema. Since \( f'' \left( \frac{2}{3} \right) \) is undefined, \( \frac{2}{3} \) may be an inflection point. Using test value 0 and 1, we find that \( f''(0) > 0 \) and \( f''(1) < 0 \), so \( \left( \frac{2}{3}, 0 \right) \) is an inflection point. The graph of \( f \) is sketched above.
4. \( f'(x) = 3x^2 + 6x = 3(x + 2). \) The critical point in \([0, 5]\) is 0. Since \( f(0) = 5 \) and \( f(5) = 125 + 75 + 5 \), the absolute maximum value is \( f(5) = 205 \) and the absolute minimum value is \( f(0) = 5 \).

5. (17 pts.)
   
   (a) (6 pts.) \( f'(x) = -(3x - 1)^{-2}(3) = \frac{3}{(3x-1)^2} \)
   
   (b) (3 pts.) \( f'(x) = 1 - \frac{1}{x} \)
   
   (c) (8 pts.) \( f''(x) = \frac{(1+\ln x) - \frac{1}{x}(x+2)}{(1+\ln x)^2} = \frac{\ln x - \frac{2}{x}}{(1+\ln x)^2} = \frac{x\ln x - 2}{x(1+\ln x)^2} \)

6. (Bonus: 5 pts.)
   
   (a) T; it should be \((-6, -41)\).
   
   (b) F; only 0 is a critical point of \( f \) since \( f'(0) \) is undefined.
   
   (c) F; \( f \) is both concave up on the left and right of \( x = 3 \).
   
   (d) F; need to find critical points first and the last step is plain wrong.
   
   (e) F; the second step should be \(-4x = -8\).

7. (Bonus: 10 pts.)

   From the figure above, \( x^2h = 768 \). So \( h = \frac{768}{x^2} \). Let \( C(x) \) be the cost of material. Then \( C(x) = 6x^2 + 2\left(4x \cdot \frac{768}{x^2}\right) = 6x^2 + \frac{6144}{x} \) with domain \((0, \infty)\). \( C'(x) = 12x - \frac{6144}{x^2} = \frac{12x^3 - 6144}{x^2} = \frac{12(x^3 - 512)}{x^2} \). Therefore 8 is the only critical point in the domain. \( C''(x) = 12 + \frac{18432}{x^3} \). Since \( C''(8) > 0 \), \( C(8) \) is an absolute minimum value of \( C \). Therefore the dimensions that will minimize the cost of materials is 8 ft \( \times \) 8 ft \( \times \) 12 ft.