It is reasonable,” he admitted. “And therefore I suspect it. Whenever you find a perfectly reasonable explanation of anything in nature or human conduct, look for something else. Things in the real universe don’t all fit together like the pieces of a child’s puzzle.”

- John Taine (Eric Temple Bell), The Time Stream, Three Science Fiction Novels

Instructions. To get full credit, you must show your work. Good luck!

1. (27 pts.) Consider the following graph of a function $f$.

![Graph of function $f$]

Determine each of the following.

(a) The domain and range.
(b) $\lim_{x \to 0^-} f(x) = f(0) = $
(c) Is $f$ continuous at $x = 0$? Differentiable at $x = 0$?
(d) $\lim_{x \to 2} f(x) = f(2) = $
(e) Is $f$ continuous at $x = 2$? Differentiable at $x = 2$?
(f) $\lim_{x \to 1} f(x) = f(1) = $
(g) Is $f$ continuous at $x = 1$? Differentiable at $x = 1$?
(h) $\lim_{x \to -7^+} f(x) = \lim_{x \to -9^-} f(x) = $
(i) True or false: $f'(-1) < 0$
2. (22 pts.) A company’s profit (in dollars) from the sale of \( x \) VCRs is approximately
\[
P(x) = \frac{10x^2 - 50x}{x + 1}
\]
(a) If they sell only 1 VCR, what is the profit? Any comment?
(b) What is the profit on the sale of 10 VCRs?
(c) Find the average rate of change of \( P \) as \( x \) changes from 1 to 10.
(d) Determine the marginal profit \( P'(x) \).
(e) Find \( P'(5) \).

3. (13 pts.) Find an equation of the tangent line to the graph of the function \( y = 8\sqrt{x} \) at the point \((1, 8)\).

4. (10 pts.) Find the limit.
   (a) \( \lim_{x \to -4} \frac{1}{(x + 4)^2} \)
   (b) \( \lim_{x \to 0} \frac{x^3 + x^2}{x^3 - x^2} \)

5. (9 pts.) Suppose that $600 is invested at 12%. How much is in the account at the end of 12 years, if interest is compounded: (a) quarterly? (b) monthly? (c) daily? Estimate to the nearest cent.

6. (10 pts.) Differentiate.
   (a) \( f(x) = \frac{x^2}{x - 1} \)
   (b) \( f(x) = -x^3 + 3x^2 + 5 \).

7. (9 pts.) Determine the domain of the functions in Problem 6.

8. (Bonus: 5 pts.) Fill in the blanks with \('T'\) if the statement is true, \('F'\) if the statement is false.
   (a) \( T \) The graph of \( g(x) = |x + 2| \) can be obtained by shifting the graph of \( y = |x| \) up 2 units.
   (b) \( F \) If \( f \) is a function of \( x \), we write \( f x \).
   (c) \( T \) \( \lim_{x \to 1} \frac{3x - 3}{x^2 - 1} = \frac{3}{2} \)
   (d) \( F \) To find \( \lim_{x \to 5} \frac{x - 5}{x^2 - 25} \), it is wrong to write the answer as \( \lim_{x \to 5} = \frac{1}{10} \).
   (e) \( T \) Given that \( f(x) = 3x^2 \), \( \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = f(x + 0) - f(x) = 0. \)

9. (Bonus: 10 pts.) For the function \( f(x) = \frac{1}{\sqrt{x}} \), find \( f'(x) \) by determining
\[
\lim_{h \to 0} \frac{f(x + h) - f(x)}{h}.
\]
(Hint: Multiply by 1, using \( \frac{\sqrt{x} + \sqrt{x + h}}{\sqrt{x} + \sqrt{x + h}} \))