Math 1190 Class Notes

1.1 Graphs and Equations

- The **graph** of an equation is a drawing that represents all ordered pairs that are solutions of the equation.

- When the essential parts of a problem are described in mathematical language, we say that we have a **mathematical model**. One model that we will use frequently throughout this course involves **compound interest**.

- **Theorem 1**: If an amount $P$ is invested at interest rate $i$, compounded annually, in $t$ years it will grow to the amount $A$ given by
  \[ A = P (1 + i)^t \]

- **Theorem 2**: If a principal $P$ is invested at interest rate $i$, compounded $n$ times a year, in $t$ years it will grow to an amount $A$ given by
  \[ A = P \left(1 + \frac{i}{n}\right)^{nt} \]

1.3 Finding Domain and Range

- **Set Notation**. The method of describing sets by listing all the members of the set is known as the **roster method**. For example, the set containing the numbers $-2, 0, \pi$ can be named $\{-2, 0, \pi\}$. To describe larger sets, we often use **set-builder notation** by specifying conditions under which an object is in a set. For example, the set of all real numbers less than 4 can be described as follows:
  \[ \{x | x \text{ is a real number less than } 4\} \]

- **Interval Notation**. If $a$ and $b$ are real numbers such that $a < b$, we define the interval $(a, b)$ as the set of all numbers between but not including $a$ and $b$. Thus, $(a, b) = \{x | a < x < b\}$. Note that the parentheses indicate that the endpoints are not included in the set. If the endpoints are included in the set, we use brackets instead. Thus, $[a, b] = \{x | a \leq x \leq b\}$. For intervals that extend without bound in one or both directions, we use the symbols $\infty$, read “infinity,” and $-\infty$, read “negative infinity,” to name these intervals.

- A set of ordered pairs is a **relation**. When a set of ordered pairs is such that no two different pairs share a common first coordinate, we have a function. The **domain** of a function is the set of all first coordinates and the **range** is the set of all second coordinates.

1.4 Slope and Linear Functions

- **Horizontal and Vertical Lines**
  - **Theorem 3**: The graph of $y = b$, or $f(x) = b$, a horizontal line, is the graph of a function. The graph of $x = a$, a vertical line, is not the graph of a function.
  - **Theorem 4**: The graph of $y = mx$, or $f(x) = mx$, is the straight line through the origin $(0,0)$ and the point $(1,m)$. The constant $m$ is called the **slope** of the line.

- The variable $y$ **varies directly as** $x$ if there is some positive constant $m$ such that $y = mx$. We also say that $y$ is **directly proportional** to $x$.

- A **linear function** is given by
  \[ y = mx + b \]
  and has a graph that is the straight line parallel to $y = mx$ with $y$-intercept $(0,b)$. The constant $m$ is called the **slope**.

- $y = mx + b$ is called the **slope-intercept equation** of a line.
• If a point \((x_1, y_1)\) is on the line \(y = mx + b\), it must follow that \(y_1 = mx_1 + b\). Subtracting the second equation from the first equation eliminates the \(b\)'s, and we have
\[
y - y_1 = m(x - x_1),
\]
which is called the **point-slope equation** of a line.

**Theorem 5**:
\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \text{change in } y \quad \text{change in } x = \text{slope of line containing points } (x_1, y_1) \text{ and } (x_2, y_2)
\]

• Slope can also be considered as an **average rate of change**.

• Applications of Linear Functions

1.5 Other Types of Functions

• A **quadratic function** \(f\) is given by
\[
f(x) = ax^2 + bx + c, \quad \text{where } a \neq 0.
\]

– The graph of \(f\) is called a **parabola**.

– It has a turning point, or **vertex**, at a point whose first coordinate is given by
\[
x = -\frac{b}{2a}.
\]

– It opens up if \(a > 0\) or opens down if \(a < 0\).

**Theorem 6**: The solutions of any quadratic equation \(ax^2 + bx + c = 0, a \neq 0\), are given by
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{(The Quadratic Formula)}
\]

• A **polynomial function** \(f\) is given by
\[
f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0,
\]
where \(n\) is a nonnegative integer and \(a_n, a_{n-1}, \ldots, a_1, a_0\), are real numbers, called the **coefficients** of the polynomial.

• Functions given by the quotient, or ratio, of two polynomials are called **rational functions**.

• \(y\) varies inversely as \(x\) if there is some positive number \(k\) such that \(y = \frac{k}{x}\). We also say that \(y\) is inversely proportional to \(x\).

• If the laws of exponents are to hold, we would have

1. \(a^{1/n} = \sqrt[n]{a}\), provided \(\sqrt[n]{a}\) is defined.
2. \(\sqrt[n]{a^m} = (a^m)^{1/n} = (a^{1/n})^m = a^{m/n}\).
3. \(a^{-m/n} = \frac{1}{a^{m/n}} = \frac{1}{\sqrt[n]{a^m}}\) \[44,46,48,50,52,56,58,60,66,70,73\]

• For a **demand function** \(D\), \(D(p)\) is the quantity \(x\) of units demanded by consumers when the price is \(p\). It is modeled by a decreasing function. For a **supply function** \(S\), \(S(p)\) is the quantity \(x\) of items that the sellers are willing to supply, or sell, at price \(p\). It is modeled by an increasing function. The point of intersection of the two curves \((p_E, x_E)\) is called the **equilibrium point**.