1. Find ALL solutions of
\[ \frac{dy}{dx} = \cos x \cos^2 y \]

2. Solve the initial value problem
\[ ty' + 2y = \sin t, \quad y(\pi/2) = 1. \]

3. Find the general solution of
\[ y' = \frac{3x + 2y}{5y^2 - 2x} \]

4. Consider a cylindrical water tank of constant cross section area \( A \). Water is pumped into the tank at a constant rate \( k \) and leaks out through a small hole of area \( a \) in the bottom of the tank. The rate at which water flows through the hole is \( \alpha a \sqrt{2gh} \), where \( h \) is the current depth of water in the tank, \( g \) is the acceleration due to gravity, and \( \alpha \) is a contraction coefficient that satisfies \( 0.5 \leq \alpha \leq 1 \).

   a) Derive a differential equation for the depth \( h \) of water in the tank at any time. Is the equation linear or nonlinear? Separable or not?
   b) Determine the equilibrium depth \( h_e \) of water, and show that it is asymptotically stable.

5. Find the general solution of
\[ y'' + 4y = \cos^2 x \]
(Hint: use a trigonometric identity!)

6. a) Solve the initial value problem
\[ 9y'' + 12y' + 4y = 0, \quad y(0) = a > 0, \quad y'(0) = -1. \]
   b) Find the value of \( a \) that separates solutions that become negative from those that are always positive.

7. A mass weighing 16 lb stretches a spring 3 in. The mass is attached to a viscous damper with a damping constant \( \gamma \).
   a) For which value of \( \gamma \) is the system critically damped?
   b) Assume that \( \gamma = 2 \) lb·s/ft. If the mass is set in motion from its equilibrium position with a downward velocity of 3 in/s, find its position \( u \) at any time \( t \). Determine when the mass first returns to its equilibrium position.
8. Find the simplest form of the general solution of
\[ y^{(4)} + 2y''' + 2y'' = 3e^t + 2te^t + e^{-t}\sin t. \]
Do NOT attempt to evaluate the unknown coefficients!!

9. Consider the system of second order(!) linear differential equations
\[
\begin{align*}
    u_1'' + 4u_1 &= u_2 \\
    u_2'' + 3u_2 &= 2u_1.
\end{align*}
\]
By solving the first equation for \( u_2 \) and substituting into the second equation, derive a fourth order differential equation for \( u_1 \). Solve the equation to determine expressions for \( u_1 \) and \( u_2 \).

10. Find the general solution of the system
\[
\begin{align*}
    x_1'(t) &= 3x_1(t) - x_2(t) \\
    x_2'(t) &= 4x_1(t) + 3x_2(t)
\end{align*}
\]
Also classify the origin as to type (node, saddle, center or spiral point) and state whether it is stable, asymptotically stable, or unstable.

11. Find the value(s) of \( \alpha \) where the behavior of the solutions of
\[
\begin{pmatrix}
    x' \\
    \alpha \\
\end{pmatrix} =
\begin{pmatrix}
    10 & -1 \\
    -4 & 1
\end{pmatrix}
\]
changes dramatically (i.e. where the status of the origin changes from asymptotically stable to unstable, from node to spiral point, etc.) Show explicitly how the behavior changes at each critical value of \( \alpha \).

12. Extra credit!! Show that, if \( n \neq 0, 1 \), the Bernoulli equation
\[
y' + p(t)y = q(t)y^n
\]
is reduced to a linear differential equation by the substitution \( v = y^{1-n} \).