1. Let $A$ and $B$ be two sets, and let $f : A \rightarrow B$ define a function from $A$ to $B$. Let $C_1$ and $C_2$ be subsets of $A$. Prove that

$$f(C_1 \cup C_2) = f(C_1) \cup f(C_2).$$
2. Let $A$ and $B$ be two sets, and let $f : A \rightarrow B$ define a function from $A$ to $B$. Let $D_1$ and $D_2$ be subsets of $B$. Prove that

$$f^{-1}(D_1 \cap D_2) = f^{-1}(D_1) \cap f^{-1}(D_2).$$

Note: In this case, $f^{-1}$ denotes the inverse image of sets, not necessarily an inverse function, as such a function may not be defined in this case.
3. Let $A$ and $B$ be two sets, and let $f : A \rightarrow B$ define a function from $A$ to $B$. Let $C$ be a subset of $A$. If $f$ is injective, show that $f^{-1}[f(C)] = C$. 
4. Let $A$ and $B$ be two sets, and let $f : A \rightarrow B$ define a function from $A$ to $B$. Let $D$ be a subset of $B$. If $f$ is surjective, show that $f[f^{-1}(D)] = D$. 
5. True or False: \( \bigcup_{k=1}^{\infty} P(\{1, 2, \ldots, k\}) = P(\mathbb{N}) \)

Here, \( P \) denotes "power set of," \( \{1, 2, \ldots, k\} \) represents the first \( k \) counting numbers, and \( \mathbb{N} \) stands for the set of natural numbers.
6. Prove that a countable union of countable sets is countable.
7. Let \( r \in \mathbb{R} \) such that \( r \neq 1 \). Show for all \( n \in \mathbb{N} \) that

\[
1 + r + r^2 + r^3 + \ldots + r^n = \frac{(1 - r^{n+1})}{(1 - r)}.
\]

If we let “\( n \to \infty \)”, what do we get? Under what circumstances do we get convergence? divergence?