SIMPLE AND COMPOUND INTEREST

Simple interest is mostly used for short-term loans and investments. It forms the basis of much of the mathematics of finance you will consider, like compound interest.

Principal and simple interest. When money is borrowed — whether it is a savings institution borrowing from you when you deposit money in your savings account or you borrowing from a lending agent — a fee is charged for the money borrowed. This fee is rent paid for the use of another's money, just as rent is paid for the use of another's house. The fee is called *interest*.

A sum of money that you deposit in a savings account or that you borrow from a lending agent, is referred to as the *principal.* Interest is usually computed as a percentage (called the *interest rate*) of the principal over a given period of time. An interest rate, unless otherwise stated, is an annual rate.

1. You borrowed $1,200 from a friend with the understanding that you will repay the money plus 6% simple interest when you are able. If you repay the loan after 1 year, how much interest would you owe? How much would you owe in total?

2. Find a formula which gives the total amount A you owe after t years. What changes in the formula if you had borrowed $2,500 instead of $1,200? if the interest rate is 4% instead of 6%?

Present and future value. In general, if a principal P is borrowed at a rate r, then after t years the borrower will owe the lender an amount A that will include the principal P plus the interest I. Since P is the amount that is borrowed now and A is the amount that must be paid back in the future, P is often referred to as the *present value* and A as the *future value.*

3. A loan of $4,000 was repaid at the end of 10 months with a check for $4,270. What is the future value and what is the present value? What annual rate of interest was charged?

4. At the end of 4 months, a check for $824 was used to repay a loan with an annual interest rate of 9%. What is the future value and what is the present value?

Simple interest formula. The total amount A you owe on a simple loan is a function of the principal P borrowed, the (annual) interest rate r, and the time t in years taken to repay the loan.

5. A principal P is borrowed at an (annual) interest rate r and repaid after t years. Find a formula which gives the interest I. Find another formula which gives the future value A. Check your formulas against some of the previous questions.

Compound interest. Suppose you deposit $2,000 in a savings account which earns 5% interest. After 1 year, you have earned $100 interest. If you leave that money in the bank, you will earn 5% interest on $2,100 in the second year. Interest paid on interest reinvested is called *compound interest.*

6. If you deposit $2,000 in a savings account which earns 5% interest compounded yearly, what is balance after 2 years? after 3 years?

7. Given that the balance after 10 years is $3257,79, what will be the balance in the next year?

Compounding quarterly. Another bank compounds interest *quarterly.* Compounding quarterly means that earned interest is paid to your accound at the of the each 3-month period and that interest as well as the principal earns interest for the next quarter.

8. If you deposit $2,000 in a savings account which earns 5% compounded quarterly, what is the balance after 1 quarter? Warning: $2,100 is not the right answer! Why?

Interest rates are generally quoted as *annual nominal rates*, or annual rate for short. If the compounding period is not equal to 1 year, you have to convert the interest rate to the *rate per compounding period.*

9. If 5% is the annual rate r, and interest is compounded quarterly, what is the rate per compounding period i?

10. What is the balance after 1 year? Compare this with an account where you earn 5% simple interest.

Compound interest formula. Suppose a principal P is earning interest compounded m times a year at an annual rate of r.

11. What is the rate per compounding period i?

The following table reveals the general pattern behind computing compound interest:

$$A = P(1 + i) \qquad\qquad \text{end of first period}$$
$$A = P(1 + i)^2 \qquad\qquad \text{end of second period}$$
$$A = P(1 + i)^3 \qquad\qquad \text{end of third period}$$
$$\vdots \qquad\qquad\qquad\qquad \vdots$$
$$A = P(1 + i)^n \qquad\qquad \text{end of } n\text{th period}$$

Use the compound interest formula to answer the next two questions.

12. If $5,000 is invested at 5% compounded monthly, what is the amount after 2 years? after 4 years?

13. If $2,000 is invested at 7% compounded annually, what is the amount after 5 years? what if it is compounded quarterly?

RELATED FORMULAS

14. Explain in words the following formula:

$$t = \frac{A - P}{Pr}$$

15. How is the formula above derived from the simple interest formula? What other formula(s) can you derive?

16. Create a problem which can be solved using the formula above, or one of the formulas you derived yourself.