Solutions homework set 1 - Section 1.6 – Problem solving – Exercises 8, 10, 18, 28

8.

A bank sells the following packets of traveler's checks: five \$20 bills, three \$50 bills, three \$100 bills, and five \$100 bills. How many different ways are there to buy \$500 in traveler's checks?

1. Understand the problem

We are to figure out the total number of ways you can make \$500 if you can buy the following packets: five \$20 bills, three \$50 bills, three \$100 bills, and five \$500 bills.

2. Devise a Plan

Each packet is worth a different value. We first figure out the value of each packet, then make a table listing all ways to make \$500.

3. Carry Out the Plan

The following table lists the worth of each package.

Packet	**Worth**
Five \$100 bills	\$500
Three \$100 bills	\$300
Three \$50 bills	\$150
Five \$20 bills	\$100

The next table lists the ways to make \$500, and the corresponding packets to buy. Use the last column to check.

Way to Make \$500	**Packets to Buy**	**Total**
\$500	Five \$100 bills	\$500
\$300 + \$200	Three \$100 bills and 2 × five \$20 bills	\$500
2 × \$150 + \$200	2 × three \$50 bills and five \$20 bills	\$500
5 × \$100	5 × five \$20	\$500

4. Looking Back

Using the last table we can check that we have indeed listed all the possible ways. This method also works if there are different packets to buy.

10.

A snail at the bottom of the well goes up 8 feet each day and slides back 4 feet at night. When will the snail reach the top of the well if it is (a) 50 feet deep, (b) 100 feet deep, (c) y feet deep?

1. Understand the Problem

The snail net gain in height after a day and night is 4 feet. As soon as the snail is within 8 feet of the rim, it climbs out and does not slide back. In a sense the problem is to find out in how many days the snail comes within 8 feet of the rim.

2. Devise a Plan

We understand the problem fairly well, because we have seen such a problem before. Use algebra to find the answer, or make a picture.

3. Carry Out the Plan

If the well is 50 feet deep, the snail needs to climb 50 ft – 8 ft = 42 ft before the last day. With a net gain of 4 ft, this takes 42 ft : 4 ft/day = 10.5 days. After 11 days, the snail is 50 ft – 11 × 4 ft = 6 ft from the rim. It escapes on the twelfth day.

If the well is 100 feet deep, the snail needs to climb 100 ft – 8 ft = 92 ft. This takes 92 ft : 4 ft/day = 23 days. It escapes on the 24th day.

If the well is y feet deep, compute $y - 8$ feet and divide this by 4, rounding up. The snail escapes on the next day.

4. Looking Back

By solving the problem in two particular instances (50 feet, 100 feet), we were able to describe a general method for solving these kind of problems. Since we did not know a specific depth, we used a letter for the depth of the well.

18.

Solve the following problem and tell which of the three strategies you used. A 10-lb bag of mixed nuts contains 20% peanuts. How many pounds of peanuts should be added to change the mixture to 80% peanuts?

1. Understand the Problem

We have the following information:

- the bag of mixed nuts weighs 10 pounds
- the bag contains 20% peanuts
- the mixture should be 80% peanuts.

The problem is how many pounds of peanuts to add.

2. Devise a Plan

We guess and check how many pounds to add. After each guess we compute the percentage of peanuts in the mixture. Currently, the bag contains 2 pounds of peanuts and 8 pounds of other nuts.

3. Carry out the Plan

Make a table to contain all the information.

Add ... lb of peanuts	**Weight**	**Peanuts**	**Other nuts**	**Percentage of peanuts**
0	10 lb	2 lb	8 lb	Exactly 20%
10	20 lb	12 lb	8 lb	Just over 50%
20	30 lb	22 lb	8 lb	Slightly less than 75%
30	40 lb	32 lb	8 lb	Exactly 80%

4. Looking Back

In some of the steps we did not need to compute all of the percentages exactly. All we needed to know was an approximation to the percentage of peanuts.

28.

There are 26 children in a class, including exactly 12 girls and 20 eight-year-olds. How many eight-year-old girls could there be?

1. Understand the Problem

At first it may seem that there could be between 0 and 12 eight-year-old girls. But there have to be exactly 20 eight-year-old children in total.

2. Devise a Plan

Find the number of boys and the number of children of other ages. Guess and check to find the minimum and maximum number of eight-year-old girls.

3. Carry Out the Plan

There are 26 – 12 = 24 boys and 26 – 20 = 6 children of other ages. Can there be 12 eight-year-old girls? Yes. How many girls can be of another age? At most 6. Is that possible? Yes.

4. Looking Back

There can be between 6 and 12 eight-year-old girls. In carrying out the plan, we found the right questions to ask ourselves to solve the problem. We did not use the number of boys after all.