1. (due Jan. 27, 2009; done) Show that X is a Banach space and $A \in \mathcal{B}_0(X)$ is invertible, then X is finite-dimensional.

2. (due Jan. 29, 2009) Show that if X is a compact metric space, then $C(X)_0 = \overline{C(X)_f}$.

3. (due Feb. 12, 2009; done) Show that the spectral radius of the Volterra operator is equal to 0.

4. (due Feb. 12, 2009) Show that every linear operator $T : \mathbb{R}^n \to \mathbb{R}^n$, $n \geq 3$, has a non-trivial invariant subspace.

5. (due Feb. 19, 2009; done) Let $A, B, T \in \mathcal{B}(X)$ be such that $TA = BT$. Show that the graph of (T) is an invariant closed vector subspace of the direct sum of A and B.

6. (due Feb. 19, 2009; done) Show that if X is not separable, and $T \in \mathcal{B}(X)$, then T has a non-trivial invariant closed vector subspace.

7. (due Feb. 26, 2009) Let X be a compact metric space. Let $C(X)$ be the Banach space of all complex-valued continuous functions defined on X. Let $\mathrm{H}_\alpha(X)$ be the vector subspace of $C(X)$ consisting of all Hölder continuous functions with exponent $\alpha > 0$. Endow $\mathrm{H}_\alpha(X)$ with the α-norm $||f||_\alpha = ||f||_\infty + V_\alpha(f)$. Show that $\mathrm{H}_\alpha(X)$ is a normed Banach space.

8. (due Feb. 26, 2009) With the notation and terminology of Problem 7, show that the closed unit ball in $\mathrm{H}_\alpha(X)$ is a compact set, when treated as a subset of $C(X)$ with the supremum norm $||\cdot||_\infty$.

9. (due Feb. 26, 2009) Let V_0^1 be the collection of all real-valued functions defined on the closed interval $[0, 1]$ and with bounded variation. Endow V_0^1 with the norm $||\cdot||$, $||f|| = ||f||_1 + V(f)$, where $||f||_1$ is the L_1-norm of f with respect to Lebesgue measure and $V(f)$ is the total variation of f. Show that V_0^1 is a normed Banach space.

10. (due Feb. 26, 2009) With the notation and terminology of Problem 9, show that the closed unit ball in V_0^1 is a compact set, when treated as a subset of L_1 with the L_1-norm with respect to Lebesgue measure.

11.(due March 10, 2009) Let $f : [a, b] \to \mathbb{R}$ be a function of bounded variation. Show that

$$|f(x)| \leq V_{[a,b]}(f) + \frac{1}{b-a}||f||_1$$

for all $x \in [a, b]$.