

1. Assigned Jan. 20, 2009:
Section 1.1:
5; 6; 12; 13; 16.
Due Jan. 27, 2009.

2. Assigned Jan. 22, 2009:
Section 1.1:
18; 19; 20; 22; 23;

Problem 1. Suppose that (Y, ρ) is a metric space and (X, δ) is a discrete metric space. Show that any function $f : X \rightarrow Y$ is continuous.

Problem 2. Suppose that (X, ρ) is a metric space and $f, g : X \rightarrow \mathbb{R}$ are continuous function. Prove that $f + g : X \rightarrow \mathbb{R}$ and $fg : X \rightarrow \mathbb{R}$ are continuous. Let $\lambda \in \mathbb{R}$. Show that $\lambda f : X \rightarrow \mathbb{R}$ is continuous.

Problem 3. Show the same as in Problem 2 if \mathbb{R} is replaced by \mathbb{R}^n . Then fg is to be replaced by the scalar product $f \cdot g : X \rightarrow \mathbb{R}$.

Due Jan. 29, 2009.

3. Assigned Feb. 03, 2009:
Section 1.2:
3; 6; 10; 11.

Problem 1. Show that if X is an uncountable set, then neither the topology of finite complement nor the topology of countable complement are metrizable.

Due Feb. 10, 2009.

4. Assigned Feb. 10, 2009:
Section 1.3:
4; 5; 8; 12; 22; 23.
Due Feb. 17, 2009.

5. Assigned Feb. 17, 2009:
Section 1.4:
6; 12; 18; 21; 25; 26; 29; 30.

Problem 1. Suppose that (X, d) is a metric space. Let $A \subset X$. Define the function $\delta_A : X \rightarrow \mathbb{R}$ by the following formula.

$$\delta_A(x) = \inf\{d(x, a) : a \in A\}.$$

Prove that

$$\overline{A} = \{x \in X : \delta_A(x) = 0\}.$$

Due Feb. 24, 2009.

6. Assigned Feb. 24, 2009:

Section 1.5:

3; 6 (closed sets); 15.

Problem 1. Let X be the collection of all real-valued continuous functions defined on the closed interval $[0, 1]$. Define the function $d : X \times X \rightarrow \mathbb{R}$ by the following formula.

$$d(f, g) = \int_0^1 |f(x) - g(x)| dx.$$

Prove that d is a metric on X .

Due March 03, 2009.

7. Assigned March 03, 2009:

Section 1.5:

1; 2.

Due March 10, 2009.

8. Assigned March 10, 2009:

Section 1.6:

2; 3; 4; 5; 6.

9. Assigned March 31, 2009:

Section 1.7:

4; 5; 9; 13; 15; 20; 22.

Due April 07, 2009.

9. Assigned April 09, 2009:

Section 2.1:

6; 8; 9; 11; 22.

Due April 20, 2009.