Applications of Linear Algebra in Economics

Input-Output and Inter-Industry Analysis

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Outline

1. Leontiff Input-Output Model
   - Consumption Matrices
   - Total Production, Internal Demand, and Final Demand
   - The Leontiff Input-Output Model

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Inter-Industry Demands

- A consumption matrix shows the quantity of inputs needed to produce one unit of a good.
- A simple consumption matrix:

$$\textit{Simplified Consumption Matrix } A = \qquad (1)$$

From \To	*Agg*	*Manu*	*Labor*
Agg	.25	.083	.2
Manu	.25	.167	.4
Labor	.125	.4167	.2

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Entries of Consumption Matrices

- The rows of the matrix represents the producing sector of the economy.
- The columns of the matrix represents the consuming sector of the economy.
- The entry a_{ij} in a general consumption matrix what percent of the total production value of sector j is spent on products from sector i.

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Total Production, Internal Demand, and Final Demand

- The Model:

$$\begin{bmatrix} \textit{Amount} \\ \textit{Produced} \\ x \end{bmatrix} = \begin{bmatrix} \textit{Internal} \\ \textit{Demand} \end{bmatrix} + \begin{bmatrix} \textit{Final} \\ \textit{Demand} \\ f \end{bmatrix} \tag{2}$$

Total Production, Internal Demand, and Final Demand

- x and f are represented as vectors.
  - f is demand from the non-producing sector of the economy.
  - x is the total amount of the product produced.
- The internal demand is equal to the consumption matrix multiplied by the total production vector

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The Math

- $$\begin{bmatrix} Amount \\ Produced \\ x \end{bmatrix} = [Cx] + \begin{bmatrix} Final \\ Demand \\ f \end{bmatrix} \quad (3)$$

- Therefore:

$$x = Cx + f \quad (4)$$

- Using the algebraic properties of R^n
- $$Ix = Cx + f \quad (5)$$

$$Ix - Cx = f \quad (6)$$

$$(I - C)x = f \quad (7)$$

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- The following theorem emerges:
- Let C be the consumption matrix for an economy, and let f the final demand. If C and f have nonnegative entries, and if C is economically feasible, then the inverse of the matrix (I-C) exists and the production vector:

$$x = (I - C)^{-1} f \tag{8}$$

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Summary: Key Points

- What the Consumption Matrix is and why it is important in economies.
- What the Leontiff Input-Output Model consists of and how the model is derived.
- Finally the Importance of $(I - C)^{-1}$.

- Outlook
  - Can be used to predict what will happen in economies when changes in:
    - Price
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