

1. LIMITS AND CONTINUITY

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- (1) Let $f(x)$ be a function, and let x_0 be in the domain of f . Define the statement " $f(x)$ is continuous at the point x_0 ".

$\lim_{x \rightarrow x_0} f(x)$ exists and is equal to $f(x_0)$.

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- (2) What is $\lim_{x \rightarrow +\infty} \frac{2x+4}{100x^2+4}$? = 0

because degree of numerator < degree of denominator

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- (3) What is $\lim_{x \rightarrow +\infty} \frac{2x^2+4}{x^2+100}$? = 2

because degree of numerator =
degree of denominator

5 (5) What is $\lim_{x \rightarrow 3} \frac{\sqrt{x}}{x^2 + 1}$?

$$= \frac{\sqrt{3}}{10}$$

5 (6) What is $\lim_{x \rightarrow 3} \frac{x^2 + 1}{(x - 3)^2}$? = $+\infty$

5 (7) What is $\lim_{x \rightarrow 3} \frac{1}{x - 3}$? = Does not exist

- 5 (2) Let $f(x) = 3x^6 - x^{-1} + 82x^2$. Find the derivative, $f'(x)$.

$$f'(x) = 18x^5 + x^{-2} + 164x$$

- 5 (3) Let $f(x) = \sqrt{4-x}$. Find the interval where $f(x)$ is differentiable, and find the formula for the derivative.

$f(x)$ is defined for $4-x \geq 0$ i.e. $x \leq 4$.

Differentiable for $x < 4$, interval $(-\infty, 4)$.

$$f'(x) = \frac{-1}{2\sqrt{4-x}}$$

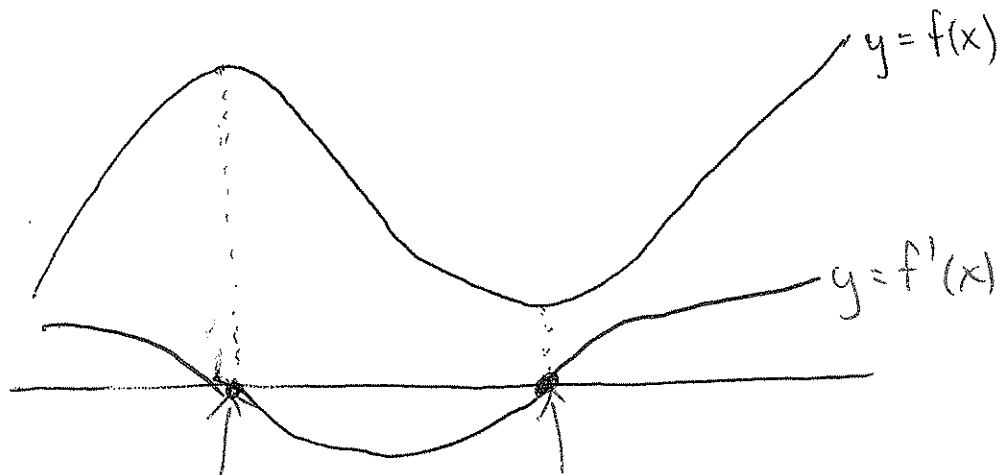
- 5 (4) Let $f(x) = \frac{\cos(x)}{x^2}$. List any points where $f(x)$ is not differentiable and find the formula for the derivative where it exists.

at $x=0$ $f(x)$ not defined, not differentiable

$$f'(x) = \frac{-x^2 \sin x - \cos(x) \cdot 2x}{x^4}$$

$$= \frac{-x \sin x - 2 \cos x}{x^3}$$

- 5 (8) (Bonus) Below is the graph of a function. On the same axes, sketch the graph of the function's derivative.



zeros of $f'(x)$ correspond to local extrema of $f(x)$

- 5 (9) (Bonus) Below is the graph of the derivative of a function. On the same axes, sketch the graph of the function.

