



- 5 (5) Let  $f(x) = \cos^2(x) \tan(x)$ . Find the derivative,  $f'(x)$ .

$$\begin{aligned} f'(x) &= \cos^2(x) \cdot \sec^2(x) + 2\cos(x)\sin(x)\tan(x) \\ &= 1 - 2\sin^2(x). \end{aligned}$$

alt:  $f(x) = \cos(x)\sin(x)$

$$f'(x) = \cos^2(x) + (-\sin^2(x)) = 1 - 2\sin^2(x)$$

- 10 (6) Find the equation of the tangent line to the graph  $y = a \cos(\pi x) + b \sin(2\pi x)$  at the point  $x = \frac{1}{6}$ .

$$y' = -a \sin(\pi x) \cdot \pi + b \cos(2\pi x) \cdot 2\pi$$

$$\text{at } x = \frac{1}{6}, y = a \cos\left(\frac{\pi}{6}\right) + b \sin\left(\frac{2\pi}{6}\right) = \frac{a\sqrt{3}}{2} + \frac{b\sqrt{3}}{2}$$

$$y' = -a\pi \sin\left(\frac{\pi}{6}\right) + 2\pi b \cos\left(\frac{2\pi}{6}\right) = -\frac{a\pi}{2} + \pi b$$

Equation:  $y - \frac{\sqrt{3}}{2}(a+b) = \pi \left(b - \frac{a}{2}\right) \left(x - \frac{1}{6}\right)$ .

- 10 (7) Find the equation of the tangent line to the graph  $y = \frac{x}{x-1}$  at the point  $x = 2$ .

$$y' = \frac{(x-1)(1) - (x)(1)}{(x-1)^2} = \frac{-1}{(x-1)^2}$$

$$\text{at } x = 2, y = \frac{2}{2-1} = 2, y' = \frac{-1}{(2-1)^2} = -1$$

Equation  $y - 2 = -1(x - 2)$

$$y = -x + 4$$

## 2. DERIVATIVES

(1) Let  $y(t) = 500 - 16t^2$ .

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(a) Calculate the average rate of change of  $y(t)$  from  $t = 1$  to  $t = 2$ . (Your answer should be a number.)

$$\frac{y(2) - y(1)}{2 - 1} = \frac{500 - 64 - (500 - 16)}{1} = -48$$

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(b) Calculate and simplify the average *Rate of change of  $y(t)$*  from  $t = 1$  to  $t = 1 + h$ , where  $h$  is an arbitrary increment of time. (Your answer should involve the symbol " $h$ ".)

$$\begin{aligned} \frac{y(1+h) - y(1)}{1+h - 1} &= \frac{500 - 16(1+h)^2 - (500 - 16)}{h} \\ &= \frac{-16(1+2h+h^2) + 16}{h} = \frac{-16(2+h)}{h} = -32 - 16h \end{aligned}$$

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(c) Calculate the instantaneous rate of change of  $y(t)$  at  $t = 1$ . (Your answer should be a number.)

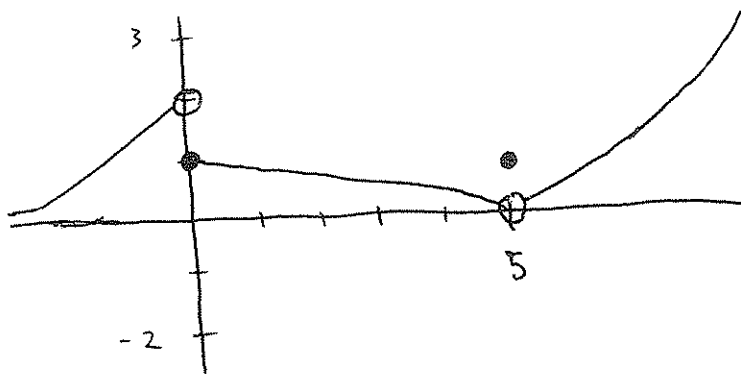
in answer to (b), take  $\lim_{h \rightarrow 0} (-32 - 16h) = -32$ .

Alternative:  $y'(t) = -32t$

$$y'(1) = -32$$

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(4) A piecewise function  $f(x)$  is graphed below.



3 (a) What is  $\lim_{x \rightarrow 0^+} f(x)$ ? = 1

3 (b) What is  $\lim_{x \rightarrow 0^-} f(x)$ ? = 2

3 (c) What is  $\lim_{x \rightarrow 0} f(x)$ ? = Does not exist

3 (d) What is  $\lim_{x \rightarrow 5} f(x)$ ? = 0

3 (e) List all the points at which  $f(x)$  is not continuous.

$$x = 0, x = 5$$