Muir, Stephen R. Gibbs/equilibrium measures for functions of multidimensional shifts with countable alphabets. Doctor of Philosophy (Mathematics), May 2011, 148 pp., 0 tables, 0 illustrations, bibliography, 16.

Consider a multidimensional shift space with a countably infinite alphabet, which serves in mathematical physics as a classical lattice gas or lattice spin system. A new definition of a Gibbs measure is introduced for suitable real-valued functions of the configuration space, which play the physical role of specific internal energy. The variational principle is proved for a large class of functions, and then a more restrictive modulus of continuity condition is provided that guarantees a function's Gibbs measures to be a nonempty, weakly compact, convex set of measures that coincides with the set of measures obeying a form of the DLR equations (which has been adapted so as to be stated entirely in terms of specific internal energy instead of the Hamiltonians for an interaction potential). The variational equilibrium measures for a such a function are then characterized as the shift invariant Gibbs measures of finite entropy, and a condition is provided to determine if a function's Gibbs measures have infinite entropy or not. Moreover the spatially averaged limiting Gibbs measures, i.e. constructive equilibria, are shown to exist and their weakly closed convex hull is shown to coincide with the set of true variational equilibrium measures. It follows that the "pure thermodynamic phases", which correspond to the extreme points in the convex set of equilibrium measures, must be constructive equilibria. Finally, for an even smoother class of functions a method is presented to construct a compatible interaction potential and it is checked that the two different structures generate the same sets of Gibbs and equilibrium measures, respectively.