MATH 1681.100 THIRD TEST

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On this page choose eight to work and cross out the other two.
Answer whether the following statements are true or false.
Please recall that the phrases "mutually exclusive" and "disjoint" have the same meaning.

(1) For any event A it is true that p(A)=1-p(not A). TRUE

(2) For any two events A and B it is true that p(A or B)=p(A)+p(B). FALSE

(3) If events A and B are mutually exclusive then p(A or B)=p(A)+p(B). TRUE

(4) If events A and B are independent then they must be mutually exclusive. FALSE

(5) If events A and B are mutually exclusive then they must be independent. FALSE

(6) If events A and B are mutually exclusive then p(A and B)=p(A)p(B). FALSE

(7) If events A and B are independent then p(A and B)=p(A)p(B). TRUE

(8) If p(A and B)=p(A)p(B) then events A and B are independent. TRUE

(9) If events A and B are mutually exclusive then p(A or B)=p(A)p(B). FALSE

(10) If events A and B are independent then p(A and B)=p(A)+p(B). FALSE

On this page choose three to work and cross out the other two.
Answer whether the given pair of events are mutually exclusive (i.e. disjoint) or not.

(1) Flipping a coin 4 times:
event A is getting 2 heads, event B is getting 2 tails.

NOT

(2) Drawing a card at random from a standard 52 card deck:
event A is drawing a heart, event B is drawing a diamond.

NOT

(3) Drawing a card at random from a standard 52 card deck:
event A is drawing a heart, event B is drawing an ace.

YES; Disjoint; Mutually Exclusive

(4) Drawing a card at random from a standard 52 card deck:
event A is drawing a face card, event B is drawing a diamond face card.

NOT

(5) Rolling a green die and a red die simultaneously:
event A is rolling a 3 on the green die, event B is rolling a 4 on the red die.

NOT

On this page choose two to work and cross out the other two.
Answer whether the given pair of events are independent or not. Justify your answer according to the definition of independence.

(1) Rolling a green die and a red die simultaneously:
event A is rolling a 3 on the green die, event B is rolling a 4 on the red die.

$P(3 \text{ on green}) = \frac{1}{6}$; $P(4 \text{ on red}) = \frac{1}{6}$; $P(A \text{ and } B) = \frac{1}{36}$.

Independent because $P(A \text{ and } B) = P(A) \cdot P(B)$.

(2) Drawing a card at random from a standard 52 card deck:
event A is drawing a heart, event B is drawing an ace.

$P(\heartsuit) = \frac{1}{4}$; $P(\text{Ace}) = \frac{4}{52} = \frac{1}{13}$

Independent.

$P(\text{Ace of } \heartsuit\text{s}) = \frac{1}{52} = \left(\frac{1}{4}\right)\left(\frac{1}{13}\right)$.

(3) Drawing a card at random from a standard 52 card deck:
event A is drawing a heart, event B is drawing a diamond.

$P(\heartsuit) = \frac{1}{4}$; $P(\diamondsuit) = \frac{1}{4}$

$P(\heartsuit \text{ and } \diamondsuit) = 0$.

Not Independent

(4) Drawing a card at random from a standard 52 card deck:
event A is drawing a face card, event B is drawing a diamond face card.

$P(\text{Face}) = \frac{12}{52} = \frac{3}{13}$; $P(\diamondsuit \text{ Face}) = \frac{3}{52}$

Not independent.

$P(\text{Face and } \diamondsuit \text{Face}) = P(\diamondsuit \text{ Face}) \neq P(\text{Face}) \cdot P(\diamondsuit \text{ Face})$.

For the remaining problems you must clearly show the steps you take to reach the answer in order to receive full credit.

(1) A jar contains 13 black beans and 47 white beans. A prisoner is made to draw a bean at random from this jar. What is the probability that he draws a black bean?

total 60 beans

$$P(\text{black bean}) = \frac{13}{60}.$$

(2) Calculate the probability of flipping four heads in a row with a fair coin.

$$\left(\frac{1}{2}\right)^4 = \frac{1}{16}.$$

(3) Simplify as much as possible: $\binom{10}{5}$, the number of ways to choose 5 objects out of 10.

$$\frac{10!}{5!\,5!} = \frac{(\cancel{10})(9)(\cancel{8}^{2})(7)(\cancel{6}^{2})}{(\cancel{5})(\cancel{4})(\cancel{3})(\cancel{2})} = (63)(4) = 252.$$

(4) Calculate the expected value for a single draw from the box $\begin{array}{ccc} 1/4 & 1/4 & 1/2 \\ \boxed{1} & \boxed{5} & \boxed{-10} \end{array}$

$$\left(\tfrac{1}{4}\right)(1) + \left(\tfrac{1}{4}\right)(5) - \left(\tfrac{1}{2}\right)(-10)$$

$$= \frac{1}{4} + \frac{5}{4} + \frac{10}{2} = \frac{6}{4} + 5 = 6\tfrac{1}{2}$$

(5) Draw a box model and state the appropriate number of draws to model the following experiment: "roll a fair die 25 times and count the number of times you roll a six".

On a single roll, $P(⚅) = \frac{1}{6}$

$\begin{array}{cc} 1/6 & 5/6 \\ \boxed{1} & \boxed{0} \end{array}$, 25 Draws

(6) Calculate the standard error for a single draw from the box $\begin{array}{cc} 1/3 & 2/3 \\ \boxed{0} & \boxed{1} \end{array}$

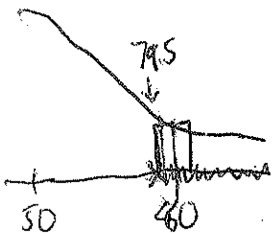
$$\sqrt{\left(\tfrac{1}{3}\right)\left(\tfrac{2}{3}\right)} = \sqrt{\frac{2}{9}} = \frac{\sqrt{2}}{3}.$$

(7) Estimate by an appropriate normal distribution (as dictated by the central limit theorem) the probability of flipping 80 or more heads in 100 flips of a fair coin.

$\boxed{1}\boxed{0}$

$$E = \left(\tfrac{1}{2}\right)(1) + \left(\tfrac{1}{2}\right)(0) = \tfrac{1}{2}$$

$$SE = \sqrt{\tfrac{1}{2}\cdot\tfrac{1}{2}} = \tfrac{1}{2}.$$



approximately normal:

100 draws: $A = (100)\left(\tfrac{1}{2}\right) = 50$

$SD = \sqrt{100}\left(\tfrac{1}{2}\right) = \tfrac{10}{2} = 5$.

$Z = \dfrac{79.5-50}{5} = \dfrac{29.5}{5} = 5.9$, % to left ≈ 100. $\boxed{\text{\% to right} \approx 0.}$

(8) Estimate by an appropriate normal distribution (as dictated by the central limit theorem) the probability that the sum of 400 draws from a box is greater than or equal to 0, given that the expected value for a single draw from the box is 0.1 and the standard error for a single draw is 2.

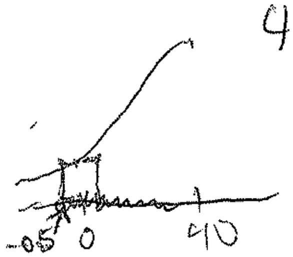
$E_{1\ \text{Draw}} = 0.1$

$SE_{1\ \text{Draw}} = 2$

approximately normal:

400 draws: $A = (400)(0.1) = 40$

$SD = \sqrt{400}\,(2) = (20)(2) = 40$



$$Z = \frac{-0.5-40}{40} = \frac{-40.5}{40} = -1.0125 \approx -1.013$$

% to left = 15.62

$\boxed{\text{\% to right} = 84.38}$

(9) Rolling 5 fair dice, calculate the probability of rolling at least 3 sixes.

$$P(\text{at least 3 ⚅es}) = P(3) + P(4) + P(5) + \cancel{P(6)}$$
$$= 1 - (P(1) + P(2) + P(0))$$

$$\binom{5}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^2 + \binom{5}{4}\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^1 + \left(\frac{1}{6}\right)^5$$

(10) Rolling 5 fair dice, calculate the probability of rolling no sixes.

$$\left(\frac{5}{6}\right)^5$$

(11) Rolling 5 fair dice, calculate the probability of rolling at least one six.

$$\boxed{1 - \left(\frac{5}{6}\right)^6}$$

Choose one of the following two "gambling" problems. Warning: These scenarios are for educational purposes only. Do not attempt to test the probabilities at home!

$1 \langle\, 2, 4, 6$

(12) A traveling carnie offers you the following game: roll a pair of fair dice and win a dollar if one die shows an even number of spots and the other shows an odd number of spots. Otherwise you lose a dollar. Draw the box model for this game and calculate your expected winnings after 10 games.

$2 \langle\, 1, 3, 5$ $3 \langle\, 2, 4, 6$

$4 \langle\, 1, 3, 5$ $5 \langle\, 2, 4, 6$

$6 \langle\, 1, 3, 5$

$P(\text{even, odd}) = \frac{18}{36} = \frac{1}{2}$ = chance of winning

$$\begin{array}{cc} \frac{1}{2} & \frac{1}{2} \\ \boxed{+1} & \boxed{-1} \end{array} \qquad E_{BOX} = 0$$

$$E_{10\text{ draws}} = 10 \cdot 0 = 0.$$

Two possible approaches, but don't focus on this one

(13) All the cow pastures in town have been paved, so Cowpoke Joe sits around playing Russian roulette with his Colt peacemaker. He places a bullet in one of the gun's six chambers and spins the cylinder around so that, as far as he knows, there is a $\frac{1}{6}$ chance of the bullet being under the hammer. If the bullet is under the hammer he's going to live 0 more years; if the bullet is not under the hammer he's going to live 32 more years. Draw a box model for Joe's game and calculate his expected number of years to live after playing 3 times (assuming he spins the cylinder again after each pull).

$$\begin{array}{cc} 1-5^3/6^3 & 5^3/6^3 \\ \boxed{0} & \boxed{32} \end{array}$$

$$E_{BOX} = 32 * \frac{5^3}{6^3}$$

$$= 18.5$$

$$\begin{array}{cc} 1/6 & 5/6 \\ \boxed{0} & \boxed{32} \end{array}$$

$$E_{BOX} = \left(\frac{1}{6}\right)(0) + \left(\frac{5}{6}\right)(32)$$

$$= \frac{80}{3} = 26.67$$

$$E_{3\text{ draws}} = 3 \cdot E_{BOX} = 80.$$