Selected Answers to Practice Final

(2) Order the set:

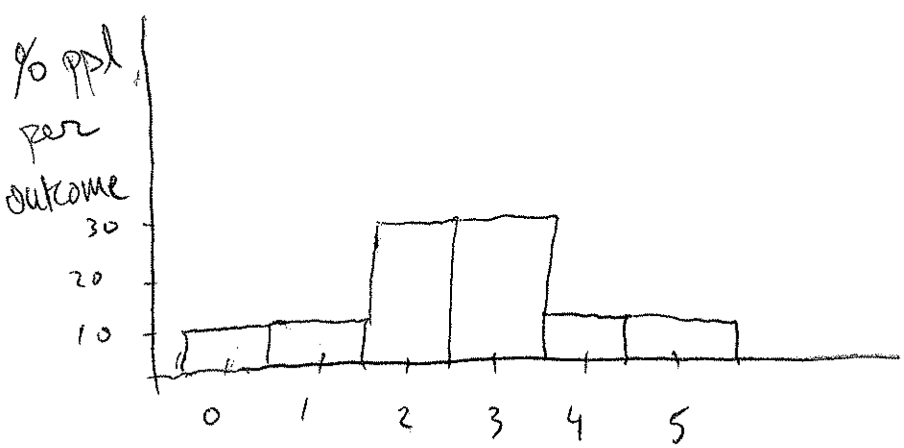
$$1, 2, 3, 4, 8, 9, 10, 20$$

anything between middle elements is median,

canonical choice is $\frac{4+8}{2} = 6$.

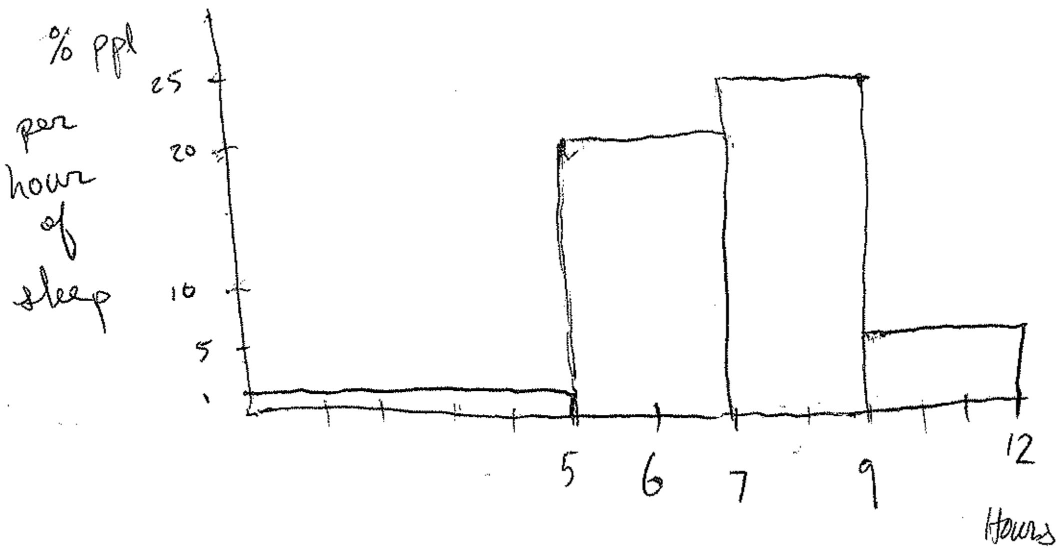
(4)

Outcome (# of Heads)	# ppl	% of ppl
0	1	10
1	1	10
2	3	30
3	3	30
4	1	10
5	1	10



(5)

Interval	# ppl	% ppl	Height of Histo. (density) (%ppl per hour)
[0, 5)	1	10	2
[5, 7)	2	20	10
[7, 9)	5	50	25
[9, 12)	2	20	6.67



(6) has one peak, symmetric falling off on either side, smooth continuous curve

(7) False

(8) How widely they are spread around the average

(9) Wider peak

(10) Area under the graph

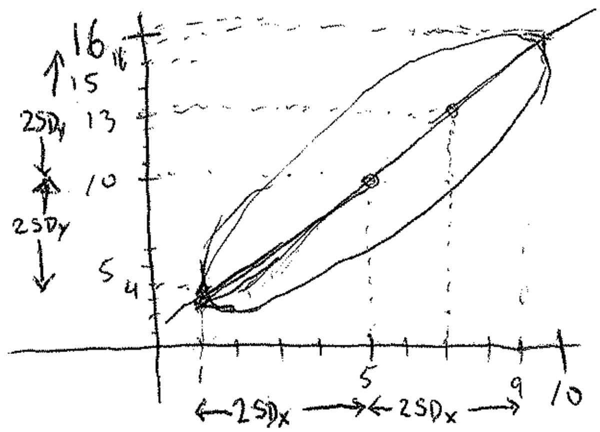
(11) SD line: $y - A_y = \frac{SD_y}{SD_x}(x - A_x)$

$$y - 10 = \frac{3}{2}(x - 5)$$

Point of averages = (5, 10)

Slope = $\frac{3}{2}$

r = 0.6, medium width oval



(16) $P(\text{event } A \text{ and } B) = P(\text{event } A) \cdot P(\text{event } B)$.

(17) event A has no outcome in common with event B,

equivalently $P(\text{event } A \text{ or } B) = P(\text{event } A) + P(\text{event } B)$.

(18) If A and B are disjoint then there are no outcomes in event A and B. Thus $P(A \text{ and } B) = 0$.

But $P(A) > 0$ and $P(B) > 0$, so $P(A) \cdot P(B) > 0$.

Thus A and B are not independent.

If A and B are independent then $P(A \text{ and } B) = P(A) \cdot P(B) > 0$.

Thus there must be an outcome in event A and B, so A and B are not disjoint.

(19) If A and B are disjoint, then $P(A \text{ or } B) = P(A) + P(B)$.

Note: If they are not disjoint, then $P(A \text{ or } B) < P(A) + P(B)$.

(20) If A and B are independent, then $P(A \text{ and } B) = P(A) \cdot P(B)$.

(21) For any event A, $P(A) + P(\text{not } A) = 1$.

Equivalently, $P(A) = 1 - P(\text{not } A)$, or $P(\text{not} A) = 1 - P(A)$.

(26) Possible totals in a pair of dice:

total:	2, 3, 4, 6, 8, 9, 10, 12
# ways to get it:	1, 2, 3, 5, 5, 4, 3, 1

$P(\text{total being a multiple of } 2) = P(\text{total being } 2 \text{ or } 4 \text{ or } 6 \text{ or } 8 \text{ or } 10 \text{ or } 12) =$

$$= \frac{1+3+5+5+3+1}{36} = \frac{18}{36} = \frac{1}{2}.$$

$P(\text{total being a multiple of } 3) = P(\text{total being } 3, 6, 9, \text{ or } 12) =$

$$= \frac{2+5+4+1}{36} = \frac{12}{36} = \frac{1}{3}.$$

$P(\text{total being a multiple of } 2 \text{ or } 3) = P(\text{total being } 2,3,4,6,8,9,10, \text{ or } 12) =$

$$= \frac{1+2+3+5+5+4+3+1}{36} = \frac{23}{36}. \quad \frac{23}{36} \neq \frac{18}{36} + \frac{12}{36}. \quad \text{FALSE}.$$

(28) P(total being 2) $= \frac{1}{36} =$ P(⚀, ⚀).

P(total being 3) $= \frac{2}{36} =$ P(⚀, ⚁ OR ⚁, ⚀)

P(total being 2 or 3) = P(⚀, ⚀ OR ⚀, ⚁ OR ⚁, ⚀) $= \frac{3}{36}$.

TRUE.

(29). TRUE

(22) FALSE

(23) FALSE

(24) TRUE

(25) TRUE

(27) TRUE

(30) $$\underbrace{\binom{20}{18}\left(\frac{1}{2}\right)^{18}\left(\frac{1}{2}\right)^{2} = \binom{20}{18}\left(\frac{1}{2}\right)^{20} = 20\,C\,18 \cdot \left(\frac{1}{2}\right)^{20}}_{\text{any is fine}}$$

(31) Ways to get total of 7 on a pair of dice:

Prob $= \frac{6}{36} = \frac{3}{18} = \frac{1}{6}$.

(32) $\binom{10}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^7$

(33) $A_{BOX} = (0.3)(5) + (0.2)(1) + (0.5)(-5) = -0.8$

$SD_{BOX}^2 = (0.3)(5-(-0.8))^2 + (0.2)(1-(-0.8))^2 + (0.5)(-5-(-0.8))^2$

$= (0.3)(5.8)^2 + (0.2)(1.8)^2 + (0.5)(-4.2)^2$

$10.092 \qquad .648 \qquad 8.82 \qquad = 19.56$

$SD_{BOX} = 4.42$

(34) $E(\text{sum of 100 draws}) = 100 \cdot E(1 \text{ draw}) = 100 \cdot A_{BOX} = (100)(-0.8)$

$= -80$.

$SE(\text{sum of 100 draws}) = \sqrt{100} \cdot SE(1 \text{ draw}) = \sqrt{100} \cdot SD_{BOX} = (10)(4.42)$

$= 44.2$

(35) $E(\text{Average of 100 draws}) = \frac{E(\text{sum of 100 draws})}{100} = -0.8$

$SE(\text{Average of 100 draws}) = \frac{SE(\text{sum of 100 draws})}{100} = .442$

(36) $A_{BOX} = (0)(0.3) + (1)(0.7) = 0.7$.

$$\underbrace{SD_{BOX} = \sqrt{(0.3)(0.7)}}_{\text{short formula for } \boxed{0}\boxed{1} \text{ Boxes}} = \sqrt{.21} \approx .46$$

(37) $E(\#\text{ 1s in 100 draws}) = (100) \cdot A_{BOX} = 100 \cdot P(1) = (100)(0.7)$
$= 70.$

$SE(\#\text{1s in 100 draws}) = \sqrt{100} \cdot SD_{BOX}$
$= (10)(.46) = 4.6$

(38) $E(\%\text{ 1s in 100 draws}) = \dfrac{E(\#\text{ 1s in 100 draws})}{100} = 0.7$ or 70%

$SE(\%\text{ 1s in 100 draws}) = \dfrac{SE(\#\text{ 1s in 100 draws})}{100} = .046$ or 4.6%

(39) $\frac{3}{20}$ $\frac{17}{20}$ $\boxed{0}\boxed{1}$

$E(\#\text{ 1s in 10 draws}) = 10 \cdot P(1) = 10 \cdot A_{BOX}$
$= \frac{17}{2} = 8.5$

$SE\left(\begin{array}{l}\#\text{ 1s in 10 draws} \\ \text{w/o replacement}\end{array}\right) = \sqrt{10} \cdot SD_{BOX} \cdot \sqrt{\dfrac{20-10}{20-1}} = \sqrt{10}\,(.357)(.725)$
$= .818$

Problems:
- asymmetric box requires several hundred in sample for normal approx.
- correction factor is significant - drawing w/o replacement $\not\to$ normal
- sample < 25 should use t-curve, not normal curve

(40) (a) Pop = 2,000. Sample 100 w/o replacement.

#1s in sample = 61.

Best estimate

Box for Pop: $\overset{.61}{\boxed{1}}\overset{.39}{\boxed{0}}$; $E(\#\ 1s \text{ in } 100 \text{ draws}) = 61$

Bootstrap → $SE\left(\#\ 1s \text{ in } 100 \text{ draws w/o replacement}\right) = \sqrt{100}\,(.488)\sqrt{\frac{2000-100}{2000-1}}$

$= 4.76$

68% C.I. 61 ± 4.76

$(56.24, 65.76)$

95% C.I. 61 ± 9.52

$(51.48, 70.52)$

99% C.I. 61 ± 14.28

$(46.72, 75.28)$

(b) $E(\%\ 1s) = .61$ or 61%

$SE(\%\ 1s) = \frac{4.76}{100} = .0476$ or 4.76%

C.I.s are same as in part (a)! (Because # draws was 100.)

(41) Null $\Rightarrow$ Box

$$\begin{array}{cc} .5 & .5 \\ \boxed{1} & \boxed{0} \\ \uparrow & \uparrow \\ \text{tail} & \text{head} \end{array}$$

$E(\#\text{ tails in 100 flips}) = 50$

$SE(\#\text{ tails in 100 flips}) = \sqrt{100}\,\sqrt{(.5)(.5)}$

$= 10(.5) = 5.$

$$Z = \frac{\overset{\text{(observed)}}{60} - \overset{\text{(Expected)}}{50}}{5} = 2.$$

% to right = 100 − % to left

= 2.5 % = P-value.

If coin is fair, then
Only 2.5 trials out of 100
should turn up ≥ 60 heads; so
Coin probably isn't fair.

$2.5\% = P < 5\%$,

$\therefore$ "statistically significant".

(with continuity correction

$$Z = \frac{59.5 - 50}{5} = 1.9,$$ slightly more to the right

still significant.)

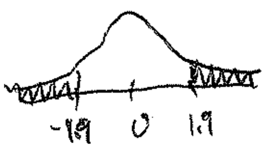
(42) Use 2-tailed z test for alternative hypothesis that coin could be biased to more or fewer tails.

$Z = 2$ (w/o continuity correction)

95% inside,
5% outside.

Borderline "significant".

$Z = 1.9$ (with continuity correction)



$>5\%$ outside. Not "significant".

Maybe coin isn't biased, afterall ???