This document will get you started using *Maple*. *Maple* is a general purpose CAS, useful for many things, but this document is geared towards linear algebra. Note that there is an extensive help facility built in to *Maple*, complete with many examples. Commands in *Maple* should in a semi-colon (normal termination) or with a colon (to suppress the output). Here's some simple examples:

$345 \cdot 57;$

$$19665 \qquad (1)$$

$357^{85};$

$$947978242027212437863247194072471926637546275641737212072343754303473276093543\backslash$$
$$236357992330546828864822091977697493178157050813474603943862833674811954911\backslash$$
$$05891380402111261862176196740318826210816627471823505190144557557 \qquad (2)$$

Note that exponentiation is entered by using the ^ symbol on the keyboard, and the right-arrow key to exit the superscript (these don't appear in the display).

Maple will always keep exact expressions unless you tell it to numerically approximate. The command evalf ("evaluate float") converts to a numerical approxinmation. An optional argument specifies how many decimal places you want (you can change the default by setting the variable Digits);

$\sin\left(\frac{2}{5}\right);$

$$\sin\left(\frac{2}{5}\right) \qquad (3)$$

$evalf\left(\sin\left(\frac{2}{5}\right)\right);$

$$0.3894183423 \qquad (4)$$

$evalf\left(\sin\left(\frac{2}{5}\right), 30\right);$

$$0.389418342308650491666311756796 \qquad (5)$$

We can assign names to expressions so that we can easily reuse them in other expressions. We use the := term ("defined equal to") to set a variable equal to another expression.

$a := 45^{31};$

$$a := 1776592919961297100543276866939850151538848876953125 \qquad (6)$$

$b := 27^{15};$

$$b := 2954312706550833698643 \qquad (7)$$

$evalf\left(\mathrm{sqrt}\left(\frac{a}{b}\right)\right);$

$$7.754713054\ 10^{14} \qquad (8)$$

$evalf\left(b^{\frac{35}{101}}\right);$

$$2.755860245\ 10^{7} \qquad \textbf{(9)}$$

A few more examples.

$c := a + b + 2;$

$$c := 177659291996129710054327686694280446424539971065177 0 \qquad \textbf{(10)}$$

$ifactor(c);$

$$(2)\ (5)\ (173)\ (1229)\ (318954253070897)\ (387360858330221466687663 83)\ (67631) \qquad \textbf{(11)}$$

$p := 2 + 31 \cdot x - 127 \cdot x^2 + x^3;$

$$p := x^3 - 127\,x^2 + 31\,x + 2 \qquad \textbf{(12)}$$

$p^3;$

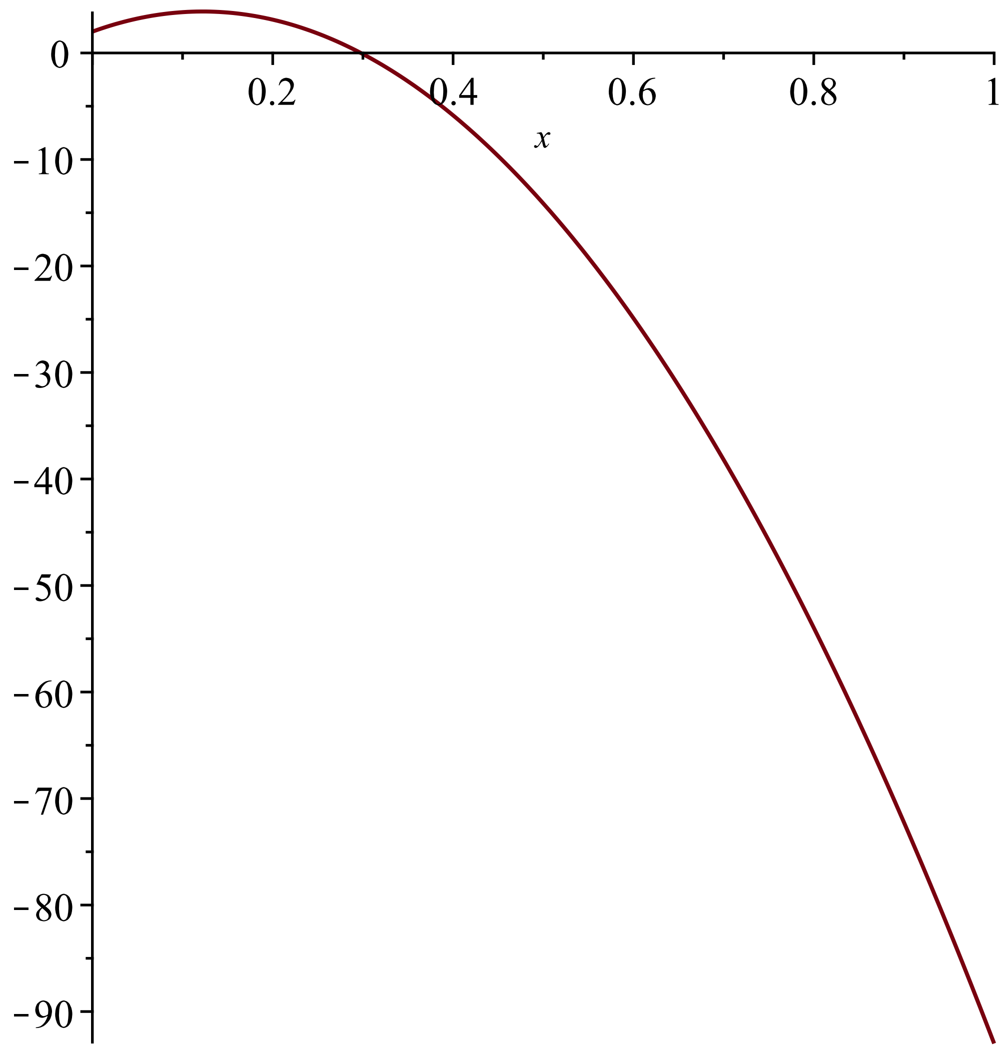
$$\left(x^3 - 127\,x^2 + 31\,x + 2\right)^3 \qquad \textbf{(13)}$$

$expand(p^3);$

$$x^9 - 381\,x^8 + 48480\,x^7 - 2071999\,x^6 + 1501356\,x^5 - 268995\,x^4 - 17441\,x^3 + 4242\,x^2 + 372\,x + 8 \qquad \textbf{(14)}$$

Before we get to linear algebra, a few example concerning plotting, which is a very powerful tool in general.

$plot(p, x = 0\,..1);$



$f := x^3 - 2 \cdot x^2 \cdot (2 - y) - y^2;$

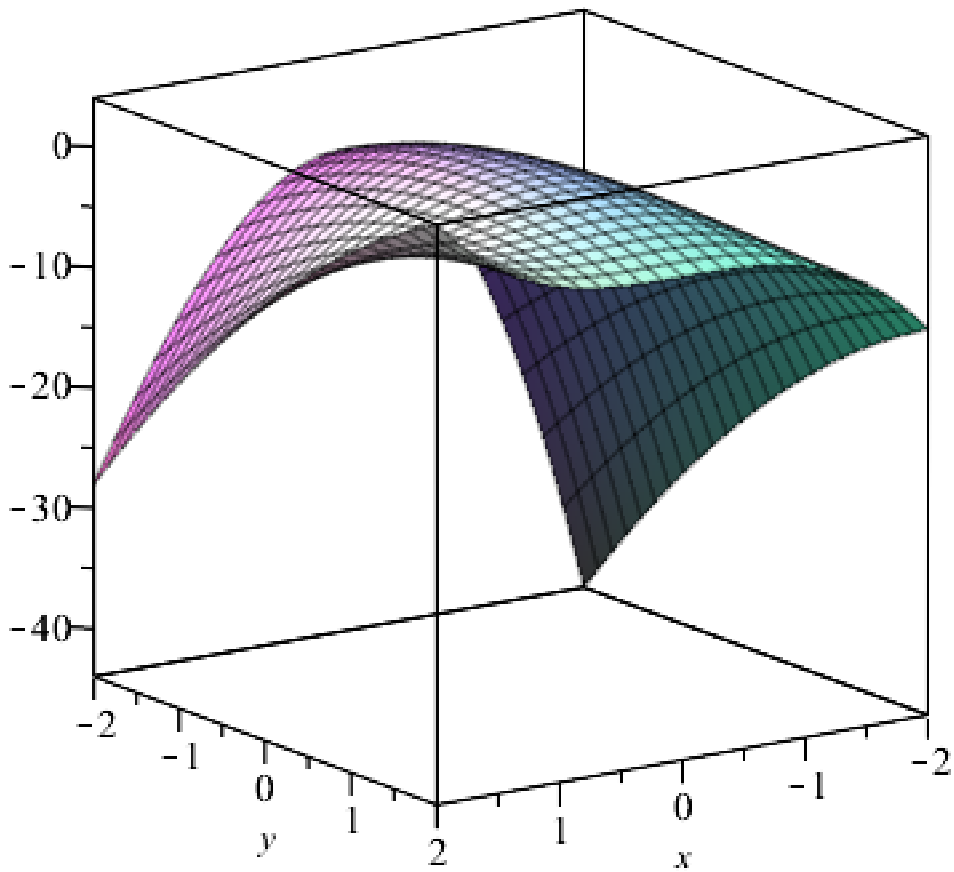
$$f := x^3 - 2\, x^2\, (2 - y) - y^2 \quad \textbf{(15)}$$

To do more specialized functions, one must sometimes read in additional packages. We read in the plots package:

with(*plots*);

[*animate*, *animate3d*, *animatecurve*, *arrow*, *changecoords*, *complexplot*, *complexplot3d*, *conformal*, *conformal3d*, *contourplot*, *contourplot3d*, *coordplot*, *coordplot3d*, *densityplot*, *display*, *dualaxisplot*, *fieldplot*, *fieldplot3d*, *gradplot*, *gradplot3d*, *implicitplot*, *implicitplot3d*, *inequal*, *interactive*, *interactiveparams*, *intersectplot*, *listcontplot*, *listcontplot3d*, *listdensityplot*, *listplot*, *listplot3d*, *loglogplot*, *logplot*, *matrixplot*, *multiple*, *odeplot*, *pareto*, *plotcompare*, *pointplot*, *pointplot3d*, *polarplot*, *polygonplot*, *polygonplot3d*, *polyhedra_supported*, *polyhedraplot*, *rootlocus*, *semilogplot*, *setcolors*, *setoptions*, *setoptions3d*, *shadebetween*, *spacecurve*, *sparsematrixplot*, *surfdata*, *textplot*, *textplot3d*, *tubeplot*] **(16)**

$plot3d(f, x=-2..2, y=-2..2, axes=boxed);$



Now we turn to linear algebra. We read in the linear algebra package first.

with(*LinearAlgebra*);

[*&x*, *Add*, *Adjoint*, *BackwardSubstitute*, *BandMatrix*, *Basis*, *BezoutMatrix*, *BidiagonalForm*, *BilinearForm*, *CARE*, *CharacteristicMatrix*, *CharacteristicPolynomial*, *Column*, *ColumnDimension*, *ColumnOperation*, *ColumnSpace*, *CompanionMatrix*, *CompressedSparseForm*, *ConditionNumber*, *ConstantMatrix*, *ConstantVector*, *Copy*, *CreatePermutation*, *CrossProduct*, *DARE*, *DeleteColumn*, *DeleteRow*, *Determinant*, *Diagonal*, *DiagonalMatrix*, *Dimension*, *Dimensions*, *DotProduct*, *EigenConditionNumbers*, *Eigenvalues*, *Eigenvectors*, *Equal*, *ForwardSubstitute*, *FrobeniusForm*, *FromCompressedSparseForm*, *FromSplitForm*, *GaussianElimination*, *GenerateEquations*, *GenerateMatrix*, *Generic*, *GetResultDataType*, *GetResultShape*, *GivensRotationMatrix*, **(17)** *GramSchmidt*, *HankelMatrix*, *HermiteForm*, *HermitianTranspose*, *HessenbergForm*, *HilbertMatrix*, *HouseholderMatrix*, *IdentityMatrix*, *IntersectionBasis*, *IsDefinite*, *IsOrthogonal*, *IsSimilar*, *IsUnitary*, *JordanBlockMatrix*, *JordanForm*, *KroneckerProduct*, *LA_Main*, *LUDecomposition*, *LeastSquares*, *LinearSolve*, *LyapunovSolve*, *Map*, *Map2*, *MatrixAdd*, *MatrixExponential*, *MatrixFunction*, *MatrixInverse*, *MatrixMatrixMultiply*, *MatrixNorm*, *MatrixPower*, *MatrixScalarMultiply*, *MatrixVectorMultiply*, *MinimalPolynomial*, *Minor*, *Modular*, *Multiply*, *NoUserValue*, *Norm*, *Normalize*, *NullSpace*, *OuterProductMatrix*, *Permanent*, *Pivot*, *PopovForm*, *ProjectionMatrix*, *QRDecomposition*, *RandomMatrix*, *RandomVector*, *Rank*, *RationalCanonicalForm*, *ReducedRowEchelonForm*, *Row*, *RowDimension*, *RowOperation*, *RowSpace*, *ScalarMatrix*, *ScalarMultiply*, *ScalarVector*, *SchurForm*, *SingularValues*, *SmithForm*, *SplitForm*, *StronglyConnectedBlocks*, *SubMatrix*, *SubVector*, *SumBasis*, *SylvesterMatrix*, *SylvesterSolve*, *ToeplitzMatrix*, *Trace*, *Transpose*, *TridiagonalForm*, *UnitVector*, *VandermondeMatrix*, *VectorAdd*, *VectorAngle*, *VectorMatrixMultiply*, *VectorNorm*, *VectorScalarMultiply*, *ZeroMatrix*, *ZeroVector*, *Zip*]

We create a few matrices.

$A := Matrix([[2, 5, 7], [-3, 8, 11], [5, 5, -6]]);$

$$A := \begin{bmatrix} 2 & 5 & 7 \\ -3 & 8 & 11 \\ 5 & 5 & -6 \end{bmatrix} \quad \textbf{(18)}$$

$B := Matrix([[1, 8, -1], [0, 3, 7], [-4, 13, 8]]);$

$$B := \begin{bmatrix} 1 & 8 & -1 \\ 0 & 3 & 7 \\ -4 & 13 & 8 \end{bmatrix} \quad \textbf{(19)}$$

Adding two matrices.

$A + B;$

$$\begin{bmatrix} 3 & 13 & 6 \\ -3 & 11 & 18 \\ 1 & 18 & 2 \end{bmatrix} \quad \textbf{(20)}$$

Multiplying two matrices. Note the use of the period . not the *

$A.B$

$$\begin{bmatrix} -26 & 122 & 89 \\ -47 & 143 & 147 \\ 29 & -23 & -18 \end{bmatrix} \quad \textbf{(21)}$$

Exponentiating a matrix.

A^5;

$$\begin{bmatrix} -11288 & 124055 & 117557 \\ -28073 & 124158 & 159461 \\ 50855 & 93555 & -64596 \end{bmatrix} \quad \textbf{(22)}$$

$A.B - B.A$;

$$\begin{bmatrix} 1 & 58 & -12 \\ -73 & 84 & 156 \\ 36 & -147 & -85 \end{bmatrix} \quad \textbf{(23)}$$

ReducedRowEchelonForm computes the reduced row echelon form of a matrix. Since that is alot to type, we introduce a new command for it:

$rref := ReducedRowEchelonForm$;

$$rref := ReducedRowEchelonForm \quad \textbf{(24)}$$

$C := rref(A)$;

$$C := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \textbf{(25)}$$

There are other ways to enter matrices as well, and many built-in operations on them.

$E := \langle \langle 3, 4, 5 \rangle \mid \langle 6, 7, 8 \rangle \mid \langle 9, 10, 11 \rangle \rangle$;

$$E := \begin{bmatrix} 3 & 6 & 9 \\ 4 & 7 & 10 \\ 5 & 8 & 11 \end{bmatrix} \quad \textbf{(26)}$$

$F := \langle A \mid E \rangle$;

$$F := \begin{bmatrix} 2 & 5 & 7 & 3 & 6 & 9 \\ -3 & 8 & 11 & 4 & 7 & 10 \\ 5 & 5 & -6 & 5 & 8 & 11 \end{bmatrix} \tag{27}$$

$G := F(1..2, 2..4);$

$$G := \begin{bmatrix} 5 & 7 & 3 \\ 8 & 11 & 4 \end{bmatrix} \tag{28}$$

$Seed := randomize(\,);$

$$Seed := 2678147411479 \tag{29}$$

$Seed := 150394368109888;$

$$Seed := 150394368109888 \tag{30}$$

$randomize(Seed);$

$$150394368109888 \tag{31}$$

We generate a random matrix with integer entries between -10 and 10.

$with(RandomTools);$

$$[AddFlavor, BlumBlumShub, Generate, GetFlavor, GetFlavors, GetState, HasFlavor, LinearCongruence, MersenneTwister, QuadraticCongruence, RandomExpand, RemoveFlavor, SetState] \tag{32}$$

$H := Matrix(5, 5, Generate(integer(range = -10..10), makeproc = true));$

$$H := \begin{bmatrix} -6 & 0 & 7 & 2 & 10 \\ 2 & 1 & 3 & 5 & 4 \\ 5 & -3 & -7 & -9 & -3 \\ -8 & -9 & -3 & -4 & 1 \\ 6 & 10 & -6 & 3 & -2 \end{bmatrix} \tag{33}$$

We do a few elementary row-operations on the matrix.

$H1 := RowOperation\left(H, 1, -\frac{1}{6}\right);$

$$H1 := \begin{bmatrix} 1 & 0 & -\frac{7}{6} & -\frac{1}{3} & -\frac{5}{3} \\ 2 & 1 & 3 & 5 & 4 \\ 5 & -3 & -7 & -9 & -3 \\ -8 & -9 & -3 & -4 & 1 \\ 6 & 10 & -6 & 3 & -2 \end{bmatrix} \tag{34}$$

$H2 := RowOperation(H1, [2, 1], -2);$

$$H2 := \begin{bmatrix} 1 & 0 & -\frac{7}{6} & -\frac{1}{3} & -\frac{5}{3} \\ 0 & 1 & \frac{16}{3} & \frac{17}{3} & \frac{22}{3} \\ 5 & -3 & -7 & -9 & -3 \\ -8 & -9 & -3 & -4 & 1 \\ 6 & 10 & -6 & 3 & -2 \end{bmatrix} \tag{35}$$

$H3 := RowOperation(H2, [3, 1], -5);$

$$H3 := \begin{bmatrix} 1 & 0 & -\frac{7}{6} & -\frac{1}{3} & -\frac{5}{3} \\ 0 & 1 & \frac{16}{3} & \frac{17}{3} & \frac{22}{3} \\ 0 & -3 & -\frac{7}{6} & -\frac{22}{3} & \frac{16}{3} \\ -8 & -9 & -3 & -4 & 1 \\ 6 & 10 & -6 & 3 & -2 \end{bmatrix} \tag{36}$$

We interchange the 4th and 5th rows (for no particular reason).

$H4 := RowOperation(H3, [4, 5]);$

$$H4 := \begin{bmatrix} 1 & 0 & -\frac{7}{6} & -\frac{1}{3} & -\frac{5}{3} \\ 0 & 1 & \frac{16}{3} & \frac{17}{3} & \frac{22}{3} \\ 0 & -3 & -\frac{7}{6} & -\frac{22}{3} & \frac{16}{3} \\ 6 & 10 & -6 & 3 & -2 \\ -8 & -9 & -3 & -4 & 1 \end{bmatrix} \tag{37}$$

$H;$

$$\begin{bmatrix} -6 & 0 & 7 & 2 & 10 \\ 2 & 1 & 3 & 5 & 4 \\ 5 & -3 & -7 & -9 & -3 \\ -8 & -9 & -3 & -4 & 1 \\ 6 & 10 & -6 & 3 & -2 \end{bmatrix} \tag{38}$$

$rref(H);$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{39}$$

We write a system of equations.

$e1 := 2 \cdot x - 3 \cdot y + 7 \cdot z = 2; e2 := -3 \cdot x + 5 \cdot y - 2 \cdot z = 3; e3 := 4 \cdot x - y + 5 \cdot z = 8;$

$$\begin{gathered} e1 := 2\,x - 3\,y + 7\,z = 2 \\ e2 := -3\,x + 5\,y - 2\,z = 3 \\ e3 := 4\,x - y + 5\,z = 8 \end{gathered} \tag{40}$$

We extract the coefficient matrix and the right-hand side vector as follows.

$A, b := GenerateMatrix(\{e1, e2, e3\}, \{x, y, z\});$

$$A, b := \begin{bmatrix} -3 & 5 & -2 \\ 2 & -3 & 7 \\ 4 & -1 & 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 8 \end{bmatrix} \tag{41}$$

If we want the augmented matrix, we use the following variation.

$Am := GenerateMatrix(\{e1, e2, e3\}, \{x, y, z\}, augmented = true);$

$$Am := \begin{bmatrix} -3 & 5 & -2 & 3 \\ 2 & -3 & 7 & 2 \\ 4 & -1 & 5 & 8 \end{bmatrix} \tag{42}$$