Review For Third Test

Test covers sections: 3.1, 3.2, 3.3, 5.1, 5.2

1.) Compute the determinant of the matrix $A = \begin{pmatrix} 2 & 7 & -1 & 9 \\ 1 & 2 & -1 & 3 \\ 2 & -2 & 0 & 2 \\ -2 & -4 & -2 & -6 \end{pmatrix}$ using only elementary row operations. Show clearly your steps.

2.) a.) Compute the cofactor matrix corresponding to the matrix $A = \begin{pmatrix} 1 & 2 & -2 \\ 2 & -1 & 3 \\ -1 & 1 & 4 \end{pmatrix}$.

b.) Compute A^{-1} using the adjoint/adgugate formula for the inverse.

3.) For which values of k is the matrix $A = \begin{pmatrix} -5 & -1 & k \\ 3 & k & 3 \\ 1 & 1 & 2 \end{pmatrix}$ invertible?

4.) a.) Compute the area of the triangle in $\mathbb{R}^2$ with vertices at $(1, 2)$, $(2, 5)$, and $(3, 10)$.

b.) The linear transformation $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + y \\ 2x - y \end{pmatrix}$ maps the triangle of part (a) onto another triangle. What is the area of the new triangle?

5.) Compute the area of the quadrilateral with vertices at $(1, 1)$, $(2, 5)$, $(6, 6)$, and $(7, 4)$.

6.) a.) Compute the volume of the parallelpiped which has one vertex at $(1, 2, -1)$ and three adjacent vertices at $(2, 3, 5)$, $(4, 7, 2)$, and $(3, 4, 9)$.

b.) Compute the volume of the tetrahedron with vertices at $(1, -1, 2)$, $(2, 1, 3)$, $(3, 4, 6)$, and $(5, 1, 3)$.

7.) Let $A = \begin{pmatrix} 2 & -1 \\ 3 & 6 \end{pmatrix}$.

a.) Compute the characteristic polynomial for the matrix A.

b.) Determine the eigenvalues of the matrix A, and for each of these eigenvalues find a corresponding eigenvector.

8.) The matrix $A = \begin{pmatrix} 6 & -1 & 2 \\ -1 & 6 & -2 \\ 2 & -2 & 9 \end{pmatrix}$ has $\lambda = 5$ as an eigenvalue. Determine a basis for the corresponding eigenspace, and say what its dimension is.

9.) Find the value(s) of k so that the vector $\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$ is an eigenvector of the matrix $A = \begin{pmatrix} 2 & 1 & -1 \\ -1 & 1 & 4 \\ 3 & -3 & k \end{pmatrix}$. Say what the corresponding eigenvalue is.

10. Determine the eigenvalues and bases for the corresponding eigenspaces for the matrix $A = \begin{pmatrix} 2 & 1 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$.

11.) Compute the determinant of the matrix $\begin{pmatrix} 3 & 1 & 2 & 1 & -1 \\ 2 & -1 & 1 & 0 & 4 \\ -2 & 1 & 3 & 0 & 1 \\ 3 & 1 & 1 & 1 & -1 \\ -1 & 2 & 1 & 0 & 6 \end{pmatrix}$ using any combination of row-reduction or cofactor expansion. Show clearly your steps.

12.) The matrix $A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$ has determinant $\det(A) = 3$. Compute the determinant of the following matrices.

a.) $\begin{pmatrix} d & e & f \\ a+2g & b+2h & c+2i \\ a & b & c \end{pmatrix}$.

b.) $\begin{pmatrix} 2a-3d & 2b-3e & 2c-3f \\ a+g & b+h & c+i \\ -g+a & -h+b & -i+c \end{pmatrix}$.

13.) If A is a square matrix and $A^2 = I$, what are the possible eigenvalues for A?