Math 2700
Review For Second Test.

1.) If $A = \begin{pmatrix} 2 & -1 \\ 3 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix}$, compute the following: A^2, AB, $(BA)^T$, $(AB)^{-1}$.

2.) Let $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by: $T(x_1, x_2, x_3) = (x_1 + 2x_2 + x_3, x_1 - x_3, 2x_2 + x_3)$.

a.) Write the standard matrix A for the linear transformation T.

b.) Write the standard matrix for $T \circ T$ (the composition of T with itself).

3.) Let $T\colon \mathbb{R}^3 \to \mathbb{R}^4$ be the linear transformation defined by: $T(x_1, x_2, x_3) = (x_1 + x_2 - x_3, 2x_1 - x_3, x_2 + x_3, x_1 + x_2 + 2x_3)$.

a.) Determine if $T(\vec{x}) = \vec{0}$ has a non-trivial solution.

b.) Determine if T is onto all of $\mathbb{R}^4$.

c.) Is the vector $\vec{x} = (1, 1, 1, -1)$ in the range of T?

4.) Suppose $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $T(\begin{bmatrix} 1 \\ 1 \end{bmatrix}) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, and $T(\begin{bmatrix} 1 \\ -1 \end{bmatrix}) = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

a.) Write $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ as a linear combination of $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

b.) Using (a), compute $T(e_1)$.

c.) Compute the standard matrix for the transformation T.

5.) Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which rotates a vector $45°$ counterclockwise and then reflects about the x-axis. Find the standard matrix for T.

6.) For each of the following matrices, determine if they are invertible, and if so find the inverse.

a.) $\begin{pmatrix} 3 & 5 \\ -2 & 4 \end{pmatrix}$.

b.) $\begin{pmatrix} -6 & 4 \\ 3 & -2 \end{pmatrix}$.

7.) For which value(s) of k is the matrix $A = \begin{pmatrix} 3 & 1 & -1 \\ 1 & 2 & 1 \\ 2 & -1 & k \end{pmatrix}$ invertible?

8.) Compute the inverse of the matrix $A = \begin{pmatrix} 1 & -3 & 2 \\ -1 & 1 & 1 \\ 3 & -6 & 1 \end{pmatrix}$ by row-reduction.

9.) Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 8 \end{pmatrix}$.

a.) Find three elementary matrices E_1, E_2, E_3 such that $E_3E_2E_1A = I$.
b.) Write A as a product of elementary matrices.

10.) Let $A = \begin{pmatrix} 1 & 2 & -3 & 2 & 0 \\ -1 & -1 & 2 & -1 & 0 \\ 2 & 3 & -5 & 2 & 0 \end{pmatrix}$.

a.) Find a basis for Null(A). What is the dimension of Null(A)?
b.) Find a basis for Col(A). What is the dimension of Col(A)?

11.) A 5×7 matrix A has 3 pivot positions.

a.) What is the dimension of Null(A)?
b.) What is the dimension of Col(A)?

12.) Find a basis for the span of the vectors $v_1 = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$, $v_2 = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$, $v_3 = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$, $v_4 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$.

13.) Show that for any square matrix A that AA^T is symmetric.

14.) Let H be the subspace of $\mathbb{R}^4$ consisting of all vectors of the form $\begin{pmatrix} a+b-c \\ a-b+3c \\ 0 \\ a+c \end{pmatrix}$.
Find a bais for H (hint: factor out the parameters to first find a set that spans H).