Math 2700
Review For First Test.

1.) Compute the reduced row-echelon form of the matrix $A = \begin{bmatrix} 1 & -2 & 2 & -1 \\ 1 & -2 & 2 & 0 \\ 1 & -2 & 3 & 2 \\ 2 & -4 & 5 & 1 \end{bmatrix}$

2.) Write the following set of equations as a matrix equation.

$$\begin{aligned} x + 3y + z &= 2 \\ -x \qquad + 2z &= 1 \\ -x + y + 2z &= 4 \end{aligned}$$

b.) Solve this set of equations.

3.) Suppose the augmented matrix of a system of equations is given by:

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 & -2 & 0 & 1 \\ 0 & 0 & 1 & 0 & 3 & 0 & 2 \\ 0 & 0 & 0 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

Write down the general solution to the system, expressing your answer in parametric form.

4.) For which values of h is the vector $\begin{bmatrix} 1 \\ 2 \\ h \end{bmatrix}$ in the span of the vectors $v_1 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$, and $v_3 = \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix}$?

b.) Determine the span of the vectors v_1, v_2, v_3.

5.) Determine if the vectors $v_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$, $v_2 = \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$, $v_3 = \begin{bmatrix} -3 \\ 4 \\ 3 \end{bmatrix}$, $v_4 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$, span $\mathbb{R}^3$.

b.) Are the vectors v_1, v_2, v_3, v_4 independent?

c.) Are the vectors v_1, v_2, v_4 independent?

6.) a.) Are the vectors $v_1 = \begin{pmatrix} -2 \\ 1 \\ 1 \\ -2 \end{pmatrix}$, $v_2 = \begin{pmatrix} 2 \\ -1 \\ -1 \\ 2 \end{pmatrix}$, $v_3 = \begin{pmatrix} -3 \\ 2 \\ 2 \\ -3 \end{pmatrix}$, $v_4 = \begin{pmatrix} 5 \\ 3 \\ 4 \\ 2 \end{pmatrix}$ independent?

b.) Do these vectors span $\mathbb{R}^4$?

7.) a.) For which value(s) of k are the vectors $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 0 \\ k \\ 3 \end{pmatrix}$, $\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$ independent?

b.) For which values(s) of k do these vectors span $\mathbb{R}^3$?

8.) Determine the equation of the plane in $\mathbb{R}^3$ which is the span of the vectors $u = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$ and $v = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$.

9.) Let $A = \begin{bmatrix} 2 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 3 & -6 \end{bmatrix}$

a.) Determine the set of vectors $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ such that $A \cdot \vec{x} = \vec{0}$.

b.) Determine the set of vectors $\vec{b}$ such that the matrix equation $A \cdot x = \vec{b}$ has a solution.

10.) If A is an $m \times n$ matrix whose columns span $\mathbb{R}^m$, how many pivot entries does A have?

What can you say about the relative sizes of m and n?

11.) Find a 3×3 matrix A such that $A \cdot \vec{x} = \vec{0}$ precisely when $x_1 + x_2 + x_3 = 0$.

12.) If $A = \begin{pmatrix} 4 & -2 & 3 \\ -1 & 5 & 2 \end{pmatrix}$, $u = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$, $v = \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}$, compute the following.

a.) $A \cdot u$.

b.) $A \cdot (3u - 2v)$.

c.) For which vectors $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3$ do we have $A \cdot x = \vec{0}$? Describe this set geometrically.

13.) If $A = \begin{pmatrix} 1 & -1 & 1 & 1 \\ 2 & -1 & 4 & 3 \\ 1 & 0 & 3 & 1 \end{pmatrix}$, find the general solution to the equation $A \cdot \vec{x} = \vec{0}$.

14.) a.) Give the equation of the line in $\mathbb{R}^3$ through the points $(2, -1, 4)$ and $(3, 1, 6)$, expressing your answer in parametric form.

b.) does this line intersect the line through the points $(1, 2, -1)$ and $(-1, 3, 2)$?