Math 2700 Review For Final
Final covers sections: 2.8, 2.9, 3.1-3.3, 5.1-5.3
Note: It would be good to review the third test as well.

Problems for sections 2.8, 2.9

1.) Let $A = \begin{pmatrix} 0 & 1 & 2 & -3 & -1 & 3 \\ 0 & 2 & 7 & 0 & -3 & 4 \\ 0 & 1 & 3 & -1 & -1 & 2 \\ 0 & 2 & 6 & -2 & -1 & 3 \end{pmatrix}$.

a.) Find the rank of A, and find the dimensions of the column space and null space of A.

b.) Find bases for the column space and null space of A.

2.) Let $A = \begin{pmatrix} 1 & 4 & 2 \\ 0 & 3 & 1 \\ 2 & -1 & 1 \end{pmatrix}$.

a.) For which value(s) of k is the vector $\begin{pmatrix} 2 \\ 1 \\ k \end{pmatrix}$ in the null space of A?

b.) For which value(s) of k is the vector $\begin{pmatrix} 2 \\ 1 \\ k \end{pmatrix}$ in the column space of A?

3.) a.) Find a subset of the vectors $v_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 1 \end{pmatrix}$, $v_2 = \begin{pmatrix} 2 \\ 4 \\ -2 \\ 2 \end{pmatrix}$, $v_3 = \begin{pmatrix} 2 \\ 1 \\ 2 \\ 1 \end{pmatrix}$ which forms a basis for $H = \text{span}\{v_1, v_2, v_3\}$.

b.) Find a subset of the vectors $w_1 = \begin{pmatrix} 0 \\ 3 \\ -4 \\ 1 \end{pmatrix}$, $w_2 = \begin{pmatrix} -1 \\ 1 \\ -3 \\ 0 \end{pmatrix}$, $w_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$, $w_4 = \begin{pmatrix} 1 \\ -1 \\ 2 \\ -3 \end{pmatrix}$ which can be added to the basis of (a) to get a basis for $\mathbb{R}^4$.

4.) Let H be the subsapce of $\mathbb{R}^3$ consisting of all vectors of the form $\begin{pmatrix} z + 2x - y \\ 2z + 4x - y \\ 4y \end{pmatrix}$. Find a basis for H.

5.) a.) Are the vectors $u = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$, $v = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$, $w = \begin{pmatrix} -1 \\ 0 \\ -5 \end{pmatrix}$ independent? Do these vectors span $\mathbb{R}^3$. Do they form a basis for $\mathbb{R}^3$?

b.) For which value(s) of k are the vectors $\{u, v, x\}$ independent, where $x = \begin{pmatrix} k \\ 1 \\ k \end{pmatrix}$? For which value(s) of k do these vectors span $\mathbb{R}^3$.

Problems from sections 3.1-3.3

6.) Let $A = \begin{pmatrix} 2 & 1 & -3 \\ 0 & 1 & 4 \\ -2 & 1 & -2 \end{pmatrix}$.

a.) Compute $\det(A)$ by cofactor expansion along the third column.
b.) Compute A^{-1} using the adjugate formula for the inverse.

7.) Let $A = \begin{pmatrix} 0 & 1 & 4 & 2 \\ 3 & -2 & 2 & 3 \\ 4 & 1 & -1 & 3 \\ 2 & -2 & 1 & 1 \end{pmatrix}$. Given that $\det(A) = -40$, compute the $(3, 2)$ entry of A^{-1}.

8.) Use any combination of elementary row/column operation and/or cofactor expansions to compute the determinant of the matrix $\begin{pmatrix} 2 & -1 & 3 & 6 \\ 4 & -1 & 5 & 10 \\ 1 & 3 & -2 & -3 \\ 8 & -4 & 6 & 12 \end{pmatrix}$. Show every step of your computation.

9.) a.) Compute the area of the quadrilateral with vertices at $p = (-2, -3)$, $q = (-1, 7)$, $r = (4, 5)$, and $s = (2, -6)$.

b.) Compute the volume of the tetrahedron with vertices at $p = (1, 1, 2)$, $q = (2, 1, 3)$, $r = (4, -1, 3)$, and $s = (0, 2, 7)$.

10.) If T is the linear transformation given by $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + 2y \\ -x + y \end{pmatrix}$, compute the area of $T(A)$ where A is the region between the circles of radius 1 and 2 centered at $(0, 0)$.

Problems for sections 5.1-5.3

11.) Is 3 an eigenvalue of the matrix $\begin{pmatrix} 5 & -1 & 3 \\ 1 & 5 & 5 \\ 2 & -6 & -1 \end{pmatrix}$? If so, find a corresponding eigenvector.

12.) For which value(s) of k is the vector $\begin{pmatrix} 1 \\ 2 \\ k \end{pmatrix}$ an eigenvector of the matrix $A = \begin{pmatrix} -3 & 1 & -1 \\ 3 & 2 & 1 \\ 5 & -4 & 1 \end{pmatrix}$?

13.) The matrix $A = \begin{pmatrix} 8 & -3 & -3 \\ -9 & 2 & 3 \\ 27 & -9 & -10 \end{pmatrix}$ has characteristic polynomial

$$p(\lambda) = -(\lambda + 1)^2(\lambda - 2).$$

Find bases for the eigenspaces and determine if the matrix A can be diagonalized.

14.) Let $A = \begin{pmatrix} 5 & -6 \\ 3 & -4 \end{pmatrix}$. Find an invertible matrix P such that $P^{-1}AP = D$ is diagonal, and say what D is.

15.) a.) Determine if the matrix $A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{pmatrix}$ can be diagonalized.

b.) Determine if the matrix $A = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{pmatrix}$ can be diagonalized.