

We create a matrix A, and then row-reduce it.

```
A = {{2, 4, -9, 10, 9, -14, 15}, {4, 8, -18, 20, 22, -34, 38}, {2, 4, -3, 4, 7, 18, -13},  
      {10, 20, -30, 35, 50, 2, 22}, {-2, -4, 6, -7, -6, -5, 3}}
```

```
Out[5]= {{2, 4, -9, 10, 9, -14, 15}, {4, 8, -18, 20, 22, -34, 38},  
         {2, 4, -3, 4, 7, 18, -13}, {10, 20, -30, 35, 50, 2, 22}, {-2, -4, 6, -7, -6, -5, 3}}
```

```
In[6]:= A // MatrixForm
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Out[6]/MatrixForm=  

$$\begin{pmatrix} 2 & 4 & -9 & 10 & 9 & -14 & 15 \\ 4 & 8 & -18 & 20 & 22 & -34 & 38 \\ 2 & 4 & -3 & 4 & 7 & 18 & -13 \\ 10 & 20 & -30 & 35 & 50 & 2 & 22 \\ -2 & -4 & 6 & -7 & -6 & -5 & 3 \end{pmatrix}$$

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In[7]:= R = RowReduce[A] // MatrixForm
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Out[7]/MatrixForm=  

$$\begin{pmatrix} 1 & 2 & 0 & \frac{1}{2} & 0 & 0 & -\frac{72}{7} \\ 0 & 0 & 1 & -1 & 0 & 0 & -\frac{27}{14} \\ 0 & 0 & 0 & 0 & 1 & 0 & \frac{19}{14} \\ 0 & 0 & 0 & 0 & 0 & 1 & -\frac{3}{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

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Now we row-reduce manually.

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In[8]:= A1 = A
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Out[8]= {{2, 4, -9, 10, 9, -14, 15}, {4, 8, -18, 20, 22, -34, 38},  
         {2, 4, -3, 4, 7, 18, -13}, {10, 20, -30, 35, 50, 2, 22}, {-2, -4, 6, -7, -6, -5, 3}}
```

```
In[9]:= A1[[1]] = (1/2) * A1[[1]]; A1 // MatrixForm
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Out[9]/MatrixForm=  

$$\begin{pmatrix} 1 & 2 & -\frac{9}{2} & 5 & \frac{9}{2} & -7 & \frac{15}{2} \\ 4 & 8 & -18 & 20 & 22 & -34 & 38 \\ 2 & 4 & -3 & 4 & 7 & 18 & -13 \\ 10 & 20 & -30 & 35 & 50 & 2 & 22 \\ -2 & -4 & 6 & -7 & -6 & -5 & 3 \end{pmatrix}$$

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```
In[10]:= A2 = A1; A2[[2]] = A2[[2]] - 4 * A2[[1]]; A2 // MatrixForm
```

```
Out[10]/MatrixForm=  

$$\begin{pmatrix} 1 & 2 & -\frac{9}{2} & 5 & \frac{9}{2} & -7 & \frac{15}{2} \\ 0 & 0 & 0 & 0 & 4 & -6 & 8 \\ 2 & 4 & -3 & 4 & 7 & 18 & -13 \\ 10 & 20 & -30 & 35 & 50 & 2 & 22 \\ -2 & -4 & 6 & -7 & -6 & -5 & 3 \end{pmatrix}$$

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In[11]:= **A3 = A2; A3[[3]] = A3[[3]] - 2 * A3[[1]]; A3 // MatrixForm**

Out[11]//MatrixForm=

$$\begin{pmatrix} 1 & 2 & -\frac{9}{2} & 5 & \frac{9}{2} & -7 & \frac{15}{2} \\ 0 & 0 & 0 & 0 & 4 & -6 & 8 \\ 0 & 0 & 6 & -6 & -2 & 32 & -28 \\ 10 & 20 & -30 & 35 & 50 & 2 & 22 \\ -2 & -4 & 6 & -7 & -6 & -5 & 3 \end{pmatrix}$$

In[12]:= **A4 = A3; A4[[4]] = A4[[4]] - 10 * A4[[1]]; A4 // MatrixForm**

Out[12]//MatrixForm=

$$\begin{pmatrix} 1 & 2 & -\frac{9}{2} & 5 & \frac{9}{2} & -7 & \frac{15}{2} \\ 0 & 0 & 0 & 0 & 4 & -6 & 8 \\ 0 & 0 & 6 & -6 & -2 & 32 & -28 \\ 0 & 0 & 15 & -15 & 5 & 72 & -53 \\ -2 & -4 & 6 & -7 & -6 & -5 & 3 \end{pmatrix}$$

In[13]:= **A5 = A4; A5[[5]] = A5[[5]] + 2 * A5[[1]]; A5 // MatrixForm**

Out[13]//MatrixForm=

$$\begin{pmatrix} 1 & 2 & -\frac{9}{2} & 5 & \frac{9}{2} & -7 & \frac{15}{2} \\ 0 & 0 & 0 & 0 & 4 & -6 & 8 \\ 0 & 0 & 6 & -6 & -2 & 32 & -28 \\ 0 & 0 & 15 & -15 & 5 & 72 & -53 \\ 0 & 0 & -3 & 3 & 3 & -19 & 18 \end{pmatrix}$$

In[14]:= **A6 = A5; A6[{{2, 3}}] = A6[{{3, 2}}]; A6 // MatrixForm**

Out[14]//MatrixForm=

$$\begin{pmatrix} 1 & 2 & -\frac{9}{2} & 5 & \frac{9}{2} & -7 & \frac{15}{2} \\ 0 & 0 & 6 & -6 & -2 & 32 & -28 \\ 0 & 0 & 0 & 0 & 4 & -6 & 8 \\ 0 & 0 & 15 & -15 & 5 & 72 & -53 \\ 0 & 0 & -3 & 3 & 3 & -19 & 18 \end{pmatrix}$$

In[15]:= **A7 = A6; A7[[2]] = (1/6) * A7[[2]]; A7 // MatrixForm**

Out[15]//MatrixForm=

$$\begin{pmatrix} 1 & 2 & -\frac{9}{2} & 5 & \frac{9}{2} & -7 & \frac{15}{2} \\ 0 & 0 & 1 & -1 & -\frac{1}{3} & \frac{16}{3} & -\frac{14}{3} \\ 0 & 0 & 0 & 0 & 4 & -6 & 8 \\ 0 & 0 & 15 & -15 & 5 & 72 & -53 \\ 0 & 0 & -3 & 3 & 3 & -19 & 18 \end{pmatrix}$$

In[16]:= **A8 = A7; A8[[1]] = A8[[1]] + (9/2) * A8[[2]]; A8[[4]] = A8[[4]] - 15 * A8[[2]];
A8[[5]] = A8[[5]] + 3 * A8[[2]]; A8 // MatrixForm**

Out[16]//MatrixForm=

$$\begin{pmatrix} 1 & 2 & 0 & \frac{1}{2} & 3 & 17 & -\frac{27}{2} \\ 0 & 0 & 1 & -1 & -\frac{1}{3} & \frac{16}{3} & -\frac{14}{3} \\ 0 & 0 & 0 & 0 & 4 & -6 & 8 \\ 0 & 0 & 0 & 0 & 10 & -8 & 17 \\ 0 & 0 & 0 & 0 & 2 & -3 & 4 \end{pmatrix}$$

In[17]:= **A9 = A8; A9[[3]] = (1/4) * A9[[3]]; A9 // MatrixForm**

Out[17]/MatrixForm=

$$\begin{pmatrix} 1 & 2 & 0 & \frac{1}{2} & 3 & 17 & -\frac{27}{2} \\ 0 & 0 & 1 & -1 & -\frac{1}{3} & \frac{16}{3} & -\frac{14}{3} \\ 0 & 0 & 0 & 0 & 1 & -\frac{3}{2} & 2 \\ 0 & 0 & 0 & 0 & 10 & -8 & 17 \\ 0 & 0 & 0 & 0 & 2 & -3 & 4 \end{pmatrix}$$

In[18]:= **A10 = A9; A10[[1]] = A10[[1]] - 3 * A10[[3]]; A10[[2]] = A10[[2]] + (1/3) * A10[[3]];
A10[[4]] = A10[[4]] - 10 * A10[[3]]; A10[[5]] = A10[[5]] - 2 * A10[[3]]; A10 // MatrixForm**

Out[18]/MatrixForm=

$$\begin{pmatrix} 1 & 2 & 0 & \frac{1}{2} & 0 & \frac{43}{2} & -\frac{39}{2} \\ 0 & 0 & 1 & -1 & 0 & \frac{29}{6} & -4 \\ 0 & 0 & 0 & 0 & 1 & -\frac{3}{2} & 2 \\ 0 & 0 & 0 & 0 & 0 & 7 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[19]:= **A11 = A10; A11[[4]] = (1/7) * A11[[4]]; A11 // MatrixForm**

Out[19]/MatrixForm=

$$\begin{pmatrix} 1 & 2 & 0 & \frac{1}{2} & 0 & \frac{43}{2} & -\frac{39}{2} \\ 0 & 0 & 1 & -1 & 0 & \frac{29}{6} & -4 \\ 0 & 0 & 0 & 0 & 1 & -\frac{3}{2} & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 & -\frac{3}{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[20]:= **A12 = A11; A12[[1]] = A12[[1]] - A12[[1, 6]] * A12[[4]];
A12[[2]] = A12[[2]] - A12[[2, 6]] * A12[[4]];
A12[[3]] = A12[[3]] - A12[[3, 6]] * A12[[4]]; A12 // MatrixForm**

Out[20]/MatrixForm=

$$\begin{pmatrix} 1 & 2 & 0 & \frac{1}{2} & 0 & 0 & -\frac{72}{7} \\ 0 & 0 & 1 & -1 & 0 & 0 & -\frac{27}{14} \\ 0 & 0 & 0 & 0 & 1 & 0 & \frac{19}{14} \\ 0 & 0 & 0 & 0 & 0 & 1 & -\frac{3}{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[21]:= **R**

Out[21]/MatrixForm=

$$\begin{pmatrix} 1 & 2 & 0 & \frac{1}{2} & 0 & 0 & -\frac{72}{7} \\ 0 & 0 & 1 & -1 & 0 & 0 & -\frac{27}{14} \\ 0 & 0 & 0 & 0 & 1 & 0 & \frac{19}{14} \\ 0 & 0 & 0 & 0 & 0 & 1 & -\frac{3}{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

We generate a matrix from some equations, and then solve them.

In[22]:= **e1 = 3 * x - 11 * y + 7 * z == 2; e2 = x - 4 * y + 8 * z == 3; e3 = 2 * x - 5 * y - 4 * z == 11;**

In[23]= **Eq = {e1, e2, e3}**

Out[23]= {3 x - 11 y + 7 z == 2, x - 4 y + 8 z == 3, 2 x - 5 y - 4 z == 11}

In[24]= **Solve[Eq, {x, y, z}]**

Out[24]= $\left\{ \left\{ x \rightarrow \frac{785}{31}, y \rightarrow \frac{225}{31}, z \rightarrow \frac{26}{31} \right\} \right\}$

In[25]= **s = Normal[CoefficientArrays[Eq, {x, y, z}]]**

Out[25]= {{-2, -3, -11}, {{3, -11, 7}, {1, -4, 8}, {2, -5, -4}}}

In[26]= **cm = s[[2]]**

Out[26]= {{3, -11, 7}, {1, -4, 8}, {2, -5, -4}}

In[27]= **cm // MatrixForm**

Out[27]/MatrixForm=

$$\begin{pmatrix} 3 & -11 & 7 \\ 1 & -4 & 8 \\ 2 & -5 & -4 \end{pmatrix}$$

We form the augmented matrix.

In[28]= **s[[1]]**

Out[28]= {-2, -3, -11}

In[29]= **am = Transpose[Append[Transpose[cm], s[[1]]]]**

Out[29]= {{3, -11, 7, -2}, {1, -4, 8, -3}, {2, -5, -4, -11}}

In[30]= **am // MatrixForm**

Out[30]/MatrixForm=

$$\begin{pmatrix} 3 & -11 & 7 & -2 \\ 1 & -4 & 8 & -3 \\ 2 & -5 & -4 & -11 \end{pmatrix}$$

In[31]= **RowReduce[am] // MatrixForm**

Out[31]/MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & -\frac{785}{31} \\ 0 & 1 & 0 & -\frac{225}{31} \\ 0 & 0 & 1 & -\frac{26}{31} \end{pmatrix}$$