Review For Second Test

Test covers sections: 2.1-2.3, 2.8, 2.9, 3.1

1.) If $A = \begin{pmatrix} 2 & 3 \\ -1 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & -2 \\ -7 & 5 \end{pmatrix}$, compute: $(A+2B)^T$, A^{-1}, and $(AB)^{-1}$.

2.) If A and B are $n \times n$ matrices and $A(A-B) = (A-B)A$, show that $A(A+B) = (A+B)A$.

3.) Determine if the matrix $A = \begin{pmatrix} 1 & 2 & -1 \\ 3 & 1 & -2 \\ 1 & 0 & -1 \end{pmatrix}$ is invertible, and if so, find its inverse using elementary row-operations.

4.) Compute a basis for the column space and for the null space of the matrix $\begin{bmatrix} 1 & 2 & 2 & 1 & 2 \\ 2 & 4 & 4 & -1 & 4 \\ -1 & -2 & 1 & 1 & -5 \end{bmatrix}$. What is the rank of this matrix?

5.) Let $T\colon \mathbb{R}^7 \to \mathbb{R}^5$ be a linear transformation with the null space of T having dimension 3. What is the dimension of the column space of the matrix associated to T? Is is possible for T to be onto?

6.) Find a basis for the subspace H of $\mathbb{R}^4$ spanned by the vectors $v_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \\ -3 \end{pmatrix}$, $v_2 = \begin{pmatrix} 2 \\ 4 \\ 2 \\ -6 \end{pmatrix}$, $v_3 = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 0 \end{pmatrix}$, $v_4 = \begin{pmatrix} 0 \\ 3 \\ 1 \\ -6 \end{pmatrix}$.

7.) Compute the dimension of the subspace H of $\mathbb{R}^4$ consisting of all vectors of the form $\begin{pmatrix} 4a+3b-c \\ a-b+3c \\ 0 \\ -2a-b-3c \end{pmatrix}$, and find a basis for H.

8.) A 4×7 matrix A has 3 pivot entries.
a.) What is the rank of A?
b.) What is the dimension of the null space of A?
c.) What is the dimension of the column space of A?

9.) Write the matrix $A = \begin{pmatrix} 3 & -1 \\ 2 & 1 \end{pmatrix}$ as a product of elementary matrices.

10.) For which value(s) of k is the matrix $\begin{pmatrix} 1 & 1 & k \\ -1 & 1 & 3 \\ 2 & 1 & 1 \end{pmatrix}$ invertible?

11.) Use cofactor expansion along the second row to compute the determinant of the matrix $\begin{pmatrix} 1 & -1 & 2 \\ -3 & -1 & 2 \\ 1 & 4 & -1 \end{pmatrix}$.

12.) Let $H = \text{span}\{\begin{pmatrix} 2 \\ 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ -1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 3 \\ 0 \end{pmatrix}\}$. Find a basis for H that contains the vector $\begin{pmatrix} -1 \\ 2 \\ -3 \\ 1 \end{pmatrix}$.