Math 2700
Review For First Test.

1.) Compute the reduced row-echelon form of the matrix $A = \begin{bmatrix} 1 & -2 & 2 & -1 \\ 1 & -2 & 2 & 0 \\ 1 & -2 & 3 & 2 \\ 2 & -4 & 5 & 1 \end{bmatrix}$

2.) Write the following set of equations as a matrix equation.

$$\begin{aligned} x + 3y + z &= 2 \\ -x \qquad + 2z &= 1 \\ -x + y + 2z &= 4 \end{aligned}$$

b.) Solve this set of equations.

3.) Suppose the augmented matrix of a system of equations is given by:

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 & -2 & 0 & 1 \\ 0 & 0 & 1 & 0 & 3 & 0 & 2 \\ 0 & 0 & 0 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

Write down the general solution to the system, expressing your answer in parametric form.

4.) For which values of h is the vector $\begin{bmatrix} 1 \\ 2 \\ h \end{bmatrix}$ in the span of the vectors $v_1 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$, and $v_3 = \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix}$?

b.) Determine the span of the vectors v_1, v_2, v_3.

5.) Determine if the vectors $v_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$, $v_2 = \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$, $v_3 = \begin{bmatrix} -3 \\ 4 \\ 3 \end{bmatrix}$, $v_4 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$, span $\mathbb{R}^3$.

b.) Are the vectors v_1, v_2, v_3, v_4 independent?

c.) Are the vectors v_1, v_2, v_4 independent?

6.) Let $A = \begin{bmatrix} 2 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 3 & -6 \end{bmatrix}$

a.) Determine the set of vectors $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ such that $A \cdot \vec{x} = \vec{0}$.

b.) Determine the set of vectors $\vec{b}$ such that the matrix equation $A \cdot x = \vec{b}$ has a solution.

7.) If A is an $m \times n$ matrix whose columns span $\mathbb{R}^m$, how many pivot entries does A have?

What can you say about the relative sizes of m and n?

8.) Find a 3×3 matrix A such that $A \cdot \vec{x} = \vec{0}$ precisely when $x_1 + x_2 + x_3 = 0$.

9.) Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be the linear transformation defined by: $T(x_1, x_2, x_3) = (x_1 + x_2 - x_3, 2x_1 - x_3, x_2 + x_3, x_1 + x_2 + 2x_3)$.

a.) Determine if $T(\vec{x}) = \vec{0}$ has a non-trivial solution.

b.) Determine if T is onto all of $\mathbb{R}^4$.

c.) Is the vector $\vec{x} = (1, 1, 1, -1)$ in the range of T?

10.) Write the standard matrix for the linear transformation from problem (10).

11.) Suppose $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $T(\begin{bmatrix} 1 \\ 1 \end{bmatrix}) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, and $T(\begin{bmatrix} 1 \\ -1 \end{bmatrix}) = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

a.) Write $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ as a linear combination of $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

b.) Using (a), compute $T(e_1)$.

c.) Compute the standard matrix for the transformation T.

12.) Let $T \colon \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which rotates a vector 45° counterclockwise and then reflects about the x-axis. Find the standard matrix for T.