Review For Final

Final covers sections: 2.8, 2.9, 3.1-3.3, 5.1-5.4

Problems for sections 2.8, 2.9

1.) Let $A = \begin{pmatrix} 0 & 1 & 2 & -3 & -1 & 3 \\ 0 & 2 & 7 & 0 & -3 & 4 \\ 0 & 1 & 3 & -1 & -1 & 2 \\ 0 & 2 & 6 & -2 & -1 & 3 \end{pmatrix}$.

a.) Find the rank of A, and find the dimensions of the column space and null space of A.

b.) Find bases for the column space and null space of A.

2.) a.) Find a subset of the vectors $v_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 1 \end{pmatrix}$, $v_2 = \begin{pmatrix} 2 \\ 4 \\ -2 \\ 2 \end{pmatrix}$, $v_3 = \begin{pmatrix} 2 \\ 1 \\ 2 \\ 1 \end{pmatrix}$ which forms a basis for $H = \text{span}\{v_1, v_2, v_3\}$.

b.) Find a subset of the vectors $w_1 = \begin{pmatrix} 0 \\ 3 \\ -4 \\ 1 \end{pmatrix}$, $w_2 = \begin{pmatrix} -1 \\ 1 \\ -3 \\ 0 \end{pmatrix}$, $w_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$, $w_4 = \begin{pmatrix} 1 \\ -1 \\ 2 \\ -3 \end{pmatrix}$ which can be added to the basis of (a) to get a basis for $\mathbb{R}^4$.

3.) Explain why 5 vector in $\mathbb{R}^4$ cannot be independent.

4.) Find a vector that is in $\text{span}\{\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}\}$ and in $\text{span}\{\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} -5 \\ 3 \\ 0 \end{pmatrix}\}$.

Problems from sections 3.1-3.3

5.) Let $A = \begin{pmatrix} 2 & 1 & -3 \\ 0 & 1 & 4 \\ -2 & 1 & -2 \end{pmatrix}$.

a.) Compute $\det(A)$ by cofactor expansion along the third column.

b.) Compute A^{-1} using the adjugate formula for the inverse.

6.) Let $A = \begin{pmatrix} 0 & 1 & 4 & 2 \\ 3 & -2 & 2 & 3 \\ 4 & 1 & -1 & 3 \\ 2 & -2 & 1 & 1 \end{pmatrix}$. Given that $\det(A) = -40$, compute the $(3,2)$ entry of A^{-1}.

7.) Use any combination of elementary row/column operation and/or cofactor expansions to compute the determinant of the matrix $\begin{pmatrix} 2 & -1 & 3 & 6 \\ 4 & -1 & 5 & 10 \\ 1 & 3 & -2 & -3 \\ 8 & -4 & 6 & 12 \end{pmatrix}$.

8.) a.) Compute the area of the quadrilateral with vertices at $p = (-2,-3)$, $q = (-1,7)$, $r = (4,5)$, and $s = (2,-6)$.

b.) Compute the volume of the tetrahedron with vertices at $p = (1,1,2)$, $q = (2,1,3)$, $r = (4,-1,3)$, and $s = (0,2,7)$.

c.) If T is the linear transformation given by $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+2y \\ -x+y \end{pmatrix}$, compute the area of $T(A)$ where A is the region between the circles of radius 1 and 2 centered at $(0,0)$.

Problems for sections 5.1-5.4

9.) Let $\mathcal{B} = \{v_1, v_2, v_3\}$, where $v_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$, $v_2 = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$, $v_3 = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$.

a.) If $[x]_{\mathcal{B}} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$, find $[x]_{\text{std}}$.

b.) If $[x]_{\text{std}} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$, find $[x]_{\mathcal{B}}$.

10.) Let $\mathcal{B}$ be the basis $\mathcal{B} = \{\begin{pmatrix} 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix}\}$, and let $\mathcal{C}$ be the basis $\mathcal{C} = \{\begin{pmatrix} 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}\}$.

a.) Compute the change of coordinates matrix from the $\mathcal{B}$ basis to the standard basis.

b.) Compute the change of coordinates matrix from the standard basis to the $\mathcal{B}$ basis.

c.) Compute the change of coordinates matrix from the $\mathcal{B}$ basis to the $\mathcal{C}$ basis.

11.) Let $\mathcal{B}$, $\mathcal{C}$ be the bases for $\mathbb{R}^2$ from problem (2) above.

a.) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation with $[T]_{\text{std}} = \begin{pmatrix} -1 & 3 \\ 2 & 5 \end{pmatrix}$, compute $[T]_{\mathcal{B}}$. You can leave your answer as a matrix product.

b.) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation with $[T]_{\mathcal{B}} = \begin{pmatrix} 1 & 1 \\ 2 & -3 \end{pmatrix}$, compute $[T]_{\mathcal{C}}$. You can leave your answer as a matrix product.

12.) Let $A = \begin{pmatrix} 17 & -10 \\ 30 & -18 \end{pmatrix}$, and $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ the corresponding linear transformation. Find a basis $\mathcal{B}$ for $\mathbb{R}^2$ with respect to which $[T]_{\mathcal{B}}$ is diagonal.

13.) For which value(s) of k is the vector $\begin{pmatrix} 1 \\ 2 \\ k \end{pmatrix}$ an eigenvector of the matrix $A = \begin{pmatrix} -3 & 1 & -1 \\ 3 & 2 & 1 \\ 5 & -4 & 1 \end{pmatrix}$?

14.) The matrix $A = \begin{pmatrix} 8 & -3 & -3 \\ -9 & 2 & 3 \\ 27 & -9 & -10 \end{pmatrix}$ has characteristic polynomial

$$p(\lambda) = -(\lambda + 1)^2(\lambda - 2).$$

Find bases for the eigenspaces and determine if the matrix A can be diagonalized.

15.) Determine if the matrix $A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{pmatrix}$ can be diagonalized.