Intro to Probability [4]

Notes on Probability:

[4] Adapted from notes developed by Kribs-Zaleta, et al. and Kirk Weller

1. If we roll a six-sided dice, then the set of possible outcomes is $\{1, 2, 3, 4, 5, 6\}$. If the die is unbiased, then the probability is $\frac{1}{6}$ of getting any particular outcome.

   Now suppose instead that we roll two unbiased six-sided dice simultaneously and consider the sum of the two results.

   (a) Determine the sample space of the possible outcomes from rolling the two-six sided dice.

   (b) Are each of the outcomes found in 1a each equally likely? If not, find the probability of each result happening.

   (c) In the chart below, list the possible outcomes and their corresponding probabilities (Note: there may be more spaces in the table than you need. A table of this type is called a *probability distribution*).

Outcome	Probability	Outcome	Probability	Outcome	Probability
				XXXXX	XXXXX

2. This problem will discuss finding the probability of certain gender distributions in families and their respective probabilities.

   (a) Discuss among your group what the probability of getting one boy and one girl in a two-child family is. If you come to more than one conclusion, make sure that everyone in the group understands each argument. Write down each of the conclusions that someone in your group has come to.

   (b) Use coins to simulate this experiment 25 times (I'll give more detail as to how to do this when you get there). Write down your results.

(c) Discuss among your group what the probability of getting two boys and two girls in a four-child family is. If you come to more than one conclusion, make sure that everyone in the group understands each argument. Write down each of the conclusions that someone in your group has come to.

(d) Use coins to simulate this experiment 25 times (I'll give more detail as to how to do this when you get there). Write down your results.

3. The *expected value* of a probabilistic experiment [with numerical outcomes] is the probabilistic average of the possible outcomes. For example, the expected value for the number of heads that come up on a single flip of an unbiased coin is $\frac{1}{2}$, because the possibilities (0 or 1 head) are equally likely, so the expected value is just their mean. Note that the expected value is *not* the "most likely outcome," but rather a probabilistic average. Moreover, in this instance, the expected value is not even a possible outcome of the experiment!

   If the outcomes are not equally likely, then the expected value is found by using a weighted average, i.e., each numerical outcome is weighted (multiplied) by the probability of its happening. The procedure for doing this is as follows: First multiply each outcome by its probability of it occurring; then add these products together. The expected value is also known as the *mean* of the experiment.

   (a) Find the expected number of pips showing for the experiment of rolling a single six-sided die.

   (b) Find the expected number of pips showing for rolling two six-sided dice and adding their results.

   (c) Find the probability distribution and expected value for the experiment of rolling two unbiased dice and multiplying their results.

(d) Now suppose that you have an biased dice, for which $P(1) = \frac{1}{2}$, i.e., there is a 50% chance that the die will turn up a 1. The probability of rolling any of the remaining numbers is equal, i.e., $P(2) = P(3) = ...$ Determine the expected value of rolling this die once.

(e) For the experiment of rolling this biased die twice and adding the results, write down the probability distribution table and then find the expected value.