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1) The exact value of $R(6,6)$ is not known, but it is known that $R(6,6) \leq$ 165. Suppose that any two people are not only either mutual friends or mutual enemies, but they also either mutually admire each other or mutually disapprove of each other. If there are 165 people at a party, show that at least one of the following cases holds:
a) There are three mutual friends who all admire one another
b) There are three mutual friends who all disapprove of one another
c) There are three mutual enemies who all admire one another
d) There are three mutual enemies who all disapprove of one another.
2) 

a) Find a finite deterministic automoton (i.e. draw its state diagram) that accepts precisely strings matching the regular expression $00^{*}(10)^{*} 1$.
b) Write pseudocode for a parser that decides if the input string matches the regular expression above.
3) Define a WFF, by recursion, to be any of the following:
a) a propositional variable $(p, q, r, \ldots)$
b) $(\neg A)$, where $A$ is a WFF
c) $(A \vee B)$, where $A$ and $B$ are WFFs
d) $(A \wedge B$, where $A$ and $B$ are WFFs
e) $(A \Longrightarrow B)$, where $A$ and $B$ are WFFs

Recall that we defined a positive WFF by the same definition, leaving out clause (a). Let us now define a restricted WFF, leaving out clauses (a) and (e).
a) Give an example of a WFF that is not a positive WFF
b) Give an example of a positive WFF that is not a restricted WFF
c) Prove or refute: If all propositional variables in a positive WFF are false, the positive WFF is false.
d) Prove or refute: If all propositional variables in a restricted WFF are false, the restricted WFF is false
5) We will not define "transmogrify". But, suppose you can directly transmogrify a vector of n things in $C n^{3}$ steps, and that given the transmogrification of two vectors, you can find the transmogrification of the concatenated vector in $n^{2}$ steps. Suggest a faster (divide-and-conquer) algo than the direct one for transmogrification, and give a big-O bound on its running time.
6) Consider the complete graph $K_{5}$ on five vertices.
a) What is the spanning tree produced by depth-first search?
b) What is the spanning tree produced by breadth-first search?

