Exercise set 3
Math 6010
3 October 2004

Just a few simple ones, to prepare for Tuesday's lecture....

1) How many open sets are there, in Baire space? Show this "constructively", by finding a way of *coding* open sets by elements of Baire space— that is, find a domain $D \subseteq \omega^\omega$ and a surjection $\pi : D \to \mathit{Opens}$. Hint: an open set is the countable union of basic open sets, and each basic open set is determined by a finite sequence of naturals, which in turn can be coded by a single natural.

2) Find a similar coding for G_δ's.

3) Show that the collection of Borel subsets of Baire space is the smallest collection of subsets, containing all the opens, that is closed under taking countable unions and countable intersections. (Note: I don't think this is true for a general topological space— if you've got time on your hands you might try to find a counterexample. It *is* true for a genreal Polish space)

4) Find a way of coding Borel sets by elements of Baire space, suggested by the answers to the first three problems. How many Borel sets are there?

5) (This is a little harder.) Given an *arbitrary* element of Baire space, how hard is it to decide whether it's one of the codes you constructed in problem (4)? That is, what is the complexity of the set of all such codes?