LIMIT REVIEW

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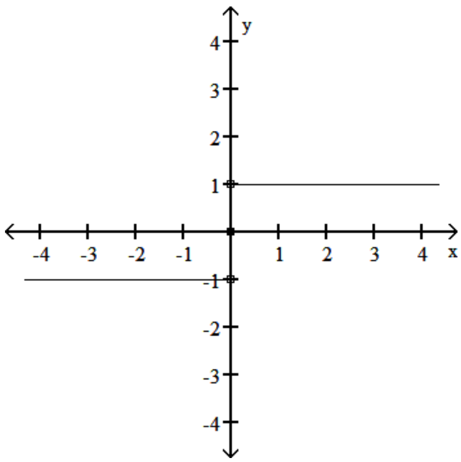
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

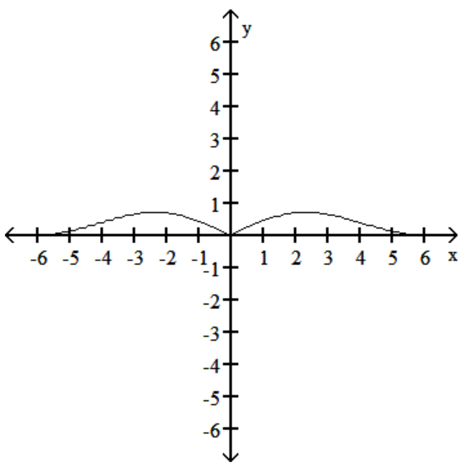
Use the graph to evaluate the limit.

1) $\lim_{x \to 0} f(x)$ 1) ________



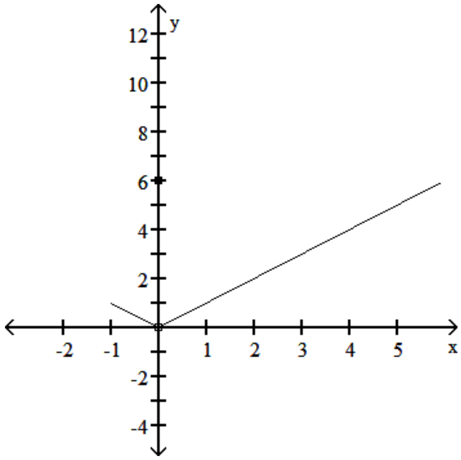
A) -1 B) ∞ C) does not exist D) 1

2) $\lim_{x \to 0} f(x)$ 2) ________



A) -1 B) 0 C) 1 D) does not exist

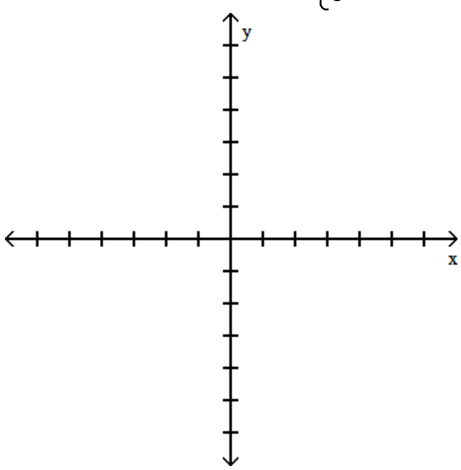
3) $\lim_{x \to 0} f(x)$ 3) ________



A) -1 B) does not exist C) 0 D) 6

Determine the limit by sketching an appropriate graph.

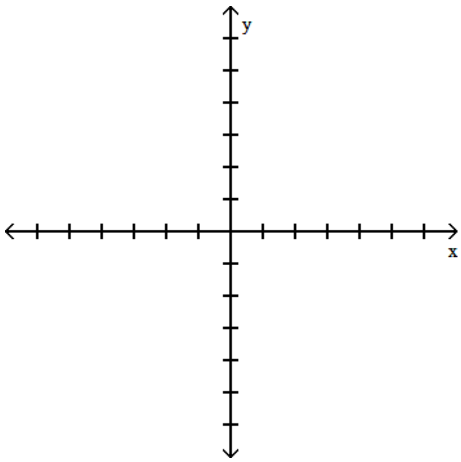
4) $\lim_{x \to 1^-} f(x)$, where $f(x) = \begin{cases} \sqrt{1 - x^2} & 0 \le x < 1 \\ 1 & 1 \le x < 3 \\ 3 & x = 3 \end{cases}$ 4) ________



A) 1 B) 3 C) 0 D) Does not exist

5) $\lim_{x \to -2^+} f(x)$, where $f(x) = \begin{cases} x^2 + 6 & \text{for } x \neq -2 \\ 0 & \text{for } x = -2 \end{cases}$

5) ________



A) 4 B) 0 C) 10 D) -2

Find numbers a and b, or k, so that f is continuous at every point.

6)

6) ________

$f(x) = \begin{cases} x^2, & x < -1 \\ ax + b, & -1 \leq x \leq 3 \\ x + 6, & x > 3 \end{cases}$

A) a = -2, b = 3 B) a = 2, b = 3 C) a = 2, b = -3 D) Impossible

7)

7) ________

$f(x) = \begin{cases} 4x + 7, & \text{if } x < -4 \\ kx + 10, & \text{if } x \geq -4 \end{cases}$

A) $k = \frac{19}{4}$ B) $k = \frac{5}{2}$ C) $k = 1$ D) $k = -\frac{5}{2}$

8)

8) ________

$f(x) = \begin{cases} 3, & x < 2 \\ ax + b, & 2 \leq x \leq 5 \\ 12, & x > 5 \end{cases}$

A) a = 3, b = 12 B) a = 3, b = -3 C) a = 3, b = 27 D) Impossible

Find the limit, if it exists.

9) $\lim_{x \to 1} \frac{x^4 - 1}{x - 1}$

9) ________

A) 2 B) 0 C) 4 D) Does not exist

10) $\lim_{x \to 0} \frac{x^3 + 12x^2 - 5x}{5x}$

10) ________

A) 5 B) Does not exist C) 0 D) -1

11) $\lim_{x \to 3} \frac{x^2 + 6x - 27}{x - 3}$ 11) _______

A) 6 B) 0 C) 12 D) Does not exist

12) $\lim_{h \to 0} \frac{(x + h)^3 - x^3}{h}$ 12) _______

A) 0 B) $3x^2 + 3xh + h^2$ C) Does not exist D) $3x^2$

13) $\lim_{x \to -6} \frac{x^2 + 12x + 36}{x + 6}$ 13) _______

A) 0 B) 144 C) 12 D) Does not exist

Find the limit.

14) $\lim_{x \to 0} \frac{x^3 - 6x + 8}{x - 2}$ 14) _______

A) 0 B) 4 C) -4 D) Does not exist

15) $\lim_{x \to 2} (x^3 + 5x^2 - 7x + 1)$ 15) _______

A) 0 B) does not exist C) 29 D) 15

16) $\lim_{x \to 1} \frac{3x^2 + 7x - 2}{3x^2 - 4x - 2}$ 16) _______

A) $-\frac{7}{4}$ B) 0 C) Does not exist D) $-\frac{8}{3}$

17) $\lim_{x \to -2} \frac{1}{x + 2}$ 17) _______

A) Does not exist B) $-\infty$ C) ∞ D) 1/2

18) $\lim_{x \to -1^-} \frac{1}{x + 1}$ 18) _______

A) $-\infty$ B) ∞ C) -1 D) 0

19) $\lim_{x \to 6^-} \frac{1}{(x - 6)^2}$ 19) _______

A) -1 B) $-\infty$ C) 0 D) ∞

20) $\lim_{x \to -\infty} \frac{5}{5 - (3/x^2)}$ 20) _______

A) $-\infty$ B) 5 C) 1 D) $\frac{5}{2}$

21) $\lim_{x \to \infty} \frac{x^2 + 6x + 7}{x^3 + 8x^2 + 4}$ 21) ______

A) $\frac{7}{4}$ B) 1 C) 0 D) ∞

22) $\lim_{x \to \infty} \frac{5x + 1}{11x - 7}$ 22) ______

A) $-\frac{1}{7}$ B) $\frac{5}{11}$ C) ∞ D) 0

23) $\lim_{x \to -\infty} \frac{6x^3 + 4x^2}{x - 7x^2}$ 23) ______

A) $-\infty$ B) $-\frac{4}{7}$ C) ∞ D) 6

24) $\lim_{x \to -\infty} \frac{\cos 4x}{x}$ 24) ______

A) 0 B) 4 C) $-\infty$ D) 1

DERIVATIVE and INTEGRATION

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Find the derivative of the function.

1) $f(t) = (6 - t)(6 + t^3)^{-1}$ 1) ________

A) $f'(t) = \frac{2t^3 - 18t^2 - 6}{(6 + t^3)^2}$

B) $f'(t) = \frac{2t^3 - 18t^2 - 6}{6 + t^3}$

C) $f'(t) = \frac{-4t^3 + 18t^2 - 6}{(6 + t^3)^2}$

D) $f'(t) = \frac{-2t^3 + 18t^2 - 6}{(6 + t^3)^2}$

2) $y = \frac{(x + 5)(x + 2)}{(x - 5)(x - 2)}$ 2) ________

A) $y' = \frac{14x - 140}{(x - 5)^2(x - 2)^2}$

B) $y' = \frac{-14x^2 + 140}{(x - 5)^2(x - 2)^2}$

C) $y' = \frac{-x^2 + 20}{(x - 5)^2(x - 2)^2}$

D) $y' = \frac{14x^2 - 140}{(x - 5)^2(x - 2)^2}$

Find the derivative.

3) $p = \frac{\sec q + \csc q}{\csc q}$ 3) ________

A) $\frac{dp}{dq} = \sec q \tan q$

B) $\frac{dp}{dq} = -\csc q \cot q$

C) $\frac{dp}{dq} = \sec^2 q$

D) $\frac{dp}{dq} = \sec^2 q + 1$

Find $\frac{d^2y}{dx^2}$ for the given function.

4) $y = 3 \cot\left(\frac{x}{10}\right)$ 4) ________

A) $-\frac{3}{10} \csc^2\left(\frac{x}{10}\right)$

B) $6 \csc^2\left(\frac{x}{10}\right) \cot\left(\frac{x}{10}\right)$

C) $\frac{3}{50} \csc^2\left(\frac{x}{10}\right) \cot\left(\frac{x}{10}\right)$

D) $-6 \csc\left(\frac{x}{10}\right)$

5) $y = (\sqrt{x} - 10)^{-3}$ 5) ________

A) $-\frac{3}{2\sqrt{x}}(\sqrt{x} - 10)^{-4}$

B) $6(\sqrt{x} - 10)^{-5}$

C) $-\frac{3}{2x}(\sqrt{x} - 10)^{-5}\left(\frac{10}{\sqrt{x}} - 3\right)$

D) $\frac{3}{4x}(\sqrt{x} - 10)^{-5}\left(-\frac{10}{\sqrt{x}} + 5\right)$

6) $y = \left(10 + \frac{4}{x}\right)^4$ 6) ________

A) $-\frac{48}{x^2}\left(10 + \frac{4}{x}\right)^2 + \frac{32}{x^3}\left(10 + \frac{4}{x}\right)^3$

B) $\frac{192}{x^4}\left(10 + \frac{4}{x}\right)^2 + \frac{32}{x^3}\left(10 + \frac{4}{x}\right)^3$

C) $12\left(10 + \frac{4}{x}\right)^2$

D) $-\frac{16}{x^2}\left(10 + \frac{4}{x}\right)^3$

At the given point, find the slope of the curve or the line that is tangent to the curve, as requested.

7) $y^6 + x^3 = y^2 + 12x$, tangent at (0, 1) 7) ________

A) $y = -\frac{3}{2}x$ B) $y = 2x + 1$ C) $y = -2x - 1$ D) $y = 3x + 1$

8) $x^4y^4 = 16$, tangent at (2, 1) 8) ________

A) $y = 8x - 1$ B) $y = -8x + 1$ C) $y = -\frac{1}{2}x + 2$ D) $y = \frac{1}{2}x$

9) $6x^2y - \pi \cos y = 7\pi$, tangent at $(1, \pi)$ 9) ________

A) $y = -2\pi x + \pi$ B) $y = \pi x$ C) $y = -2\pi x + 3\pi$ D) $y = -\frac{\pi}{2}x + \frac{3\pi}{2}$

10) $\int x \cos (2x^2)\ dx$, 10) ________

A) $\frac{1}{4}\sin (2x^2) + C$

B) $\frac{x^2}{2}\sin (2x^2) + C$

C) $\frac{1}{u}\sin (u) + C$

D) $\sin(2x^2) + C$

11) $\int \left(4 - \sin \frac{t}{2}\right)^2 \cos \frac{t}{2}\ dt$, 11) ________

A) $\frac{2}{3}\left(4 - \cos \frac{t}{2}\right)^3 + C$

B) $2\left(4 - \sin \frac{t}{2}\right)^3 + C$

C) $-\frac{2}{3}\left(4 - \sin \frac{t}{2}\right)^3 + C$

D) $\frac{1}{3}\left(4 - \sin \frac{t}{2}\right)^3 \sin \frac{t}{2} + C$

12) $\int x^4(x^5 - 9)^3\ dx$, 12) ________

A) $\frac{1}{4}(x^5 - 9)^4 + C$ B) $\frac{1}{20}x^{20} - 9 + C$ C) $\frac{1}{10}(x^5 - 9)^2 + C$ D) $\frac{1}{20}(x^5 - 9)^4 + C$

13) $\int \frac{8s^3\, ds}{\sqrt{5 - s^4}}$, 13) ______

A) $-4\sqrt{5 - s^4} + C$

B) $-4s^3\sqrt{5 - s^4} + C$

C) $\frac{4s^4}{\sqrt{5 - s^4}}$

D) $\frac{-2}{2\sqrt{5 - s^4}} + C$

14) $\int 21(y^6 + 2y^3 + 4)^3(2y^5 + 2y^2)\, dy$, 14) ______

A) $21(y^6 + 2y^3 + 4)^2 + C$

B) $\frac{7}{4}(y^6 + 2y^3 + 4)^4 + C$

C) $\frac{21}{4}(y^6 + 2y^3 + 4)^4 + C$

D) $\frac{21}{4}(y^6 + 2y^3 + 4)^4(10y^4 + 4y) + C$

15) $\int \csc^2 6\theta \cot 6\theta \, d\theta$, 15) ______

A) $\frac{1}{6} \csc^3 6\theta \cot^2 6\theta + C$

B) $-\frac{1}{12} \tan^2 6\theta + C$

C) $-\frac{1}{12} \cot^2 6\theta + C$

D) $\frac{1}{12} \cot^2 \theta + C$

16) $\int_0^{\pi} (1 + \cos 5t)^2 \sin 5t\, dt$ 16) ______

A) $\frac{1}{5}$

B) $\frac{8}{3}$

C) $\frac{1}{15}$

D) $\frac{8}{15}$

17) $\int_0^1 \frac{6\, r\, dr}{\sqrt{16 + 3r^2}}$ 17) ______

A) $2\sqrt{19} - 8$

B) $-2\sqrt{19} + 8$

C) $\frac{\sqrt{19}}{2} - 2$

D) $\sqrt{19} - 4$

18) $\int_1^4 \frac{9 - \sqrt{x}}{\sqrt{x}}\, dx$ 18) ______

A) 15

B) $-\frac{15}{2}$

C) 30

D) $\frac{15}{2}$

Find the derivative.

19) $\frac{d}{dx} \int_0^{x^3} \sin t\, dt$ 19) ______

A) $3x^2 \sin (x^3)$

B) $\sin (x^3)$

C) $-\cos (x^3) - 1$

D) $\frac{1}{4}x^4 \sin (x^3)$

20) $y = \int_0^{\tan x} \sqrt{t}\, dt$ 20) ______

A) $\sec x \tan^{3/2} x$
B) $\sec^2 x \sqrt{\tan x}$
C) $\sqrt{\tan x}$
D) $\frac{2}{3} \tan^{3/2} x$

21) $\frac{d}{dt} \int_0^{\sin t} \frac{1}{25 - u^2}\, du$ 21) ______

A) $\frac{\cos t}{25 - \sin^2 t}$
B) $\frac{1}{\cos t\,(25 - \sin^2 t)}$
C) $\frac{1}{25 - \sin^2 t}$
D) $\frac{-\cos t}{25 - \sin^2 t}$

Graph the integrand and use geometry to evaluate the integral.

22) $\int_{-4}^{4} \sqrt{16 - x^2}\, dx$ 22) ______

A) 16π
B) 16
C) 8π
D) 4π

23) $\int_{-8}^{8} (8 - |x|)\, dx$ 23) ______

A) 192
B) 32
C) 128
D) 64

24) $\int_{-4}^{2} (-2x + 4)\, dx$ 24) ______

A) 72
B) 36
C) 48
D) 12

Solve the problem.

25) Suppose that f and g are continuous and that $\int_6^{10} f(x)\, dx = -3$ and $\int_6^{10} g(x)\, dx = 9$. 25) ______

Find $\int_6^{10} [4f(x) + g(x)]\, dx$.

A) 24
B) 33
C) -3
D) 13

Use symmetry to evaluate the integral.

26) $\int_{-\pi/2}^{\pi/2} x^5 + x^3 + 18x + \sin(10x) \quad dx$ 26) ______

A) $-\frac{2}{15}$
B) 0
C) $\frac{1}{15}$
D) $\frac{1}{4}$

27) $\int_{-\pi/2}^{\pi/2} \sin^5 3x \, dx$ 27) ______

A) $\frac{8}{45}$ B) $-\frac{16}{45}$ C) 0 D) $\frac{2}{3}$

APPLICATION

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The equation gives the position s = f(t) of a body moving on a coordinate line (s in meters, t in seconds).

1) s = 6 sin t - cos t 1) ________

Find the body's acceleration at time t = $\pi/4$ sec.

A) $\frac{7\sqrt{2}}{2}$ m/sec^2 B) $-\frac{5\sqrt{2}}{2}$ m/sec^2 C) $-\frac{7\sqrt{2}}{2}$ m/sec^2 D) $\frac{5\sqrt{2}}{2}$ m/sec^2

E) $\frac{5\sqrt{2}}{2}$ m/sec^2 F) $\frac{5\sqrt{2}}{2}$ m/sec^2 G) $\frac{5\sqrt{2}}{2}$ m/sec^2 H) $\frac{5\sqrt{2}}{2}$ m/sec^2

The function s = f(t) gives the position of a body moving on a coordinate line, with s in meters and t in seconds.

2) $s = 5t^2 + 3t + 7,\ 0 \le t \le 2$ 2) ________

Find the body's speed and acceleration at the end of the time interval.

A) 13 m/sec, 2 m/sec^2 B) 30 m/sec, 10 m/sec^2

C) 23 m/sec, 20 m/sec^2 D) 23 m/sec, 10 m/sec^2

Solve the problem.

3) Given the acceleration, initial velocity, and initial position of a body moving along a coordinate line at time t, find the body's position at time t. 3) ________

a = 18 cos 3t, v(0) = 9, s(0) = -1

A) s = 2 sin 3t + 9t - 1 B) s = 2 cos 3t - 9t - 1

C) s = - 2 sin 3t + 9t - 1 D) s = - 2 cos 3t + 9t - 1

4) A company knows that the unit cost C and the unit revenue R from the production and sale of x units are related by $C = \frac{R^2}{206{,}000} + 11{,}849$. Find the rate of change of unit revenue when the unit cost is changing by $8/unit and the unit revenue is $4000. 4) ________

A) $695.45/unit B) $1184.90/unit C) $160.00/unit D) $206.00/unit

Solve the problem. Round your answer, if appropriate.

5) One airplane is approaching an airport from the north at 122 km/hr. A second airplane approaches from the east at 238 km/hr. Find the rate at which the distance between the planes changes when the southbound plane is 36 km away from the airport and the westbound plane is 17 km from the airport. 5) ________

A) -212 km/hr B) -424 km/hr C) -318 km/hr D) -106 km/hr

Solve the problem.

6) Water is falling on a surface, wetting a circular area that is expanding at a rate of 2 mm^2/s. How fast is the radius of the wetted area expanding when the radius is 174 mm? (Round your answer to four decimal places.) 6) ________

A) 0.0115 mm/s B) 0.0018 mm/s C) 0.0037 mm/s D) 546.6367 mm/s

7) If the price charged for a candy bar is p(x) cents, then x thousand candy bars will be sold in a certain city, where $p(x) = 124 - \frac{x}{16}$. How many candy bars must be sold to maximize revenue? 7) ________

A) 992 candy bars B) 992 thousand candy bars

C) 1984 candy bars D) 1984 thousand candy bars

8) Find the optimum number of batches (to the nearest whole number) of an item that should be produced annually (in order to minimize cost) if 300,000 units are to be made, it costs $2 to store a unit for one year, and it costs $440 to set up the factory to produce each batch. 8) ________

A) 28 batches B) 20 batches C) 26 batches D) 18 batches

9) Find the number of units that must be produced and sold in order to yield the maximum profit, given the following equations for revenue and cost: 9) ________

$R(x) = 30x - 0.5x^2$

$C(x) = 10x + 3.$

A) 40 units B) 23 units C) 20 units D) 21 units

10) Suppose a business can sell x gadgets for $p = 250 - 0.01x$ dollars apiece, and it costs the business $c(x) = 1000 + 25x$ dollars to produce the x gadgets. Determine the production level and cost per gadget required to maximize profit. 10) ________

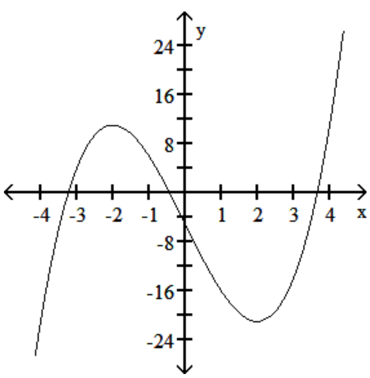
A) 11,250 gadgets at $137.50 each B) 10,000 gadgets at $150.00 each

C) 111 gadgets at $248.89 each D) 13,750 gadgets at $112.50 each

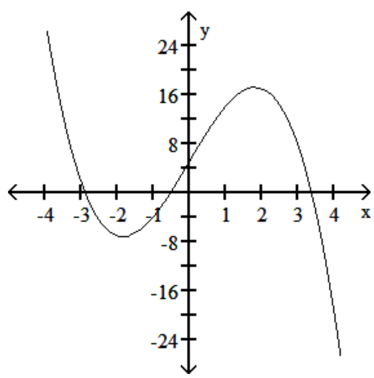
11) Using the following properties of a twice-differentiable function $y = f(x)$, select a possible graph of f 11) ________

x	y	Derivatives
$x < 2$		$y' > 0, y'' < 0$
-2	11	$y' = 0, y'' < 0$
$-2 < x < 0$		$y' < 0, y'' < 0$
0	-5	$y' < 0, y'' = 0$
$0 < x < 2$		$y' < 0, y'' > 0$
2	-21	$y' = 0, y'' > 0$
$x > 2$		$y' > 0, y'' > 0$

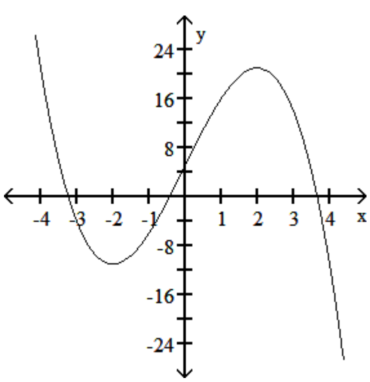
A)



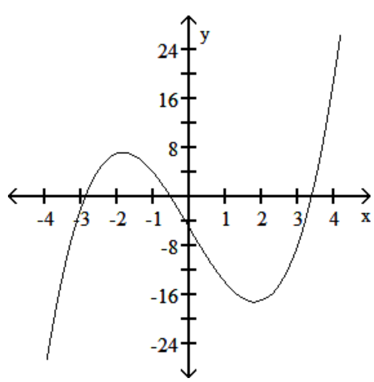
B)



C)

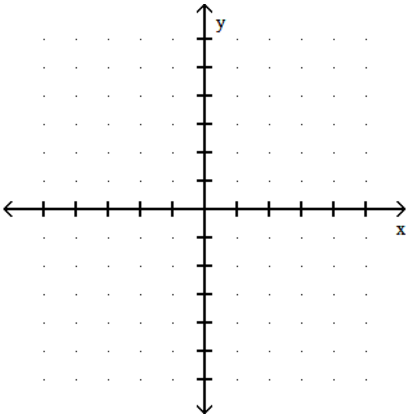


D)

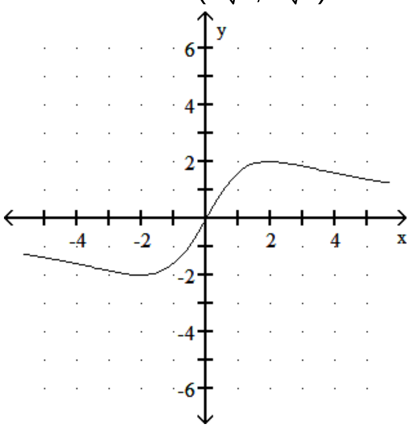


Graph the equation. Include the coordinates of any local extreme points and inflection points.

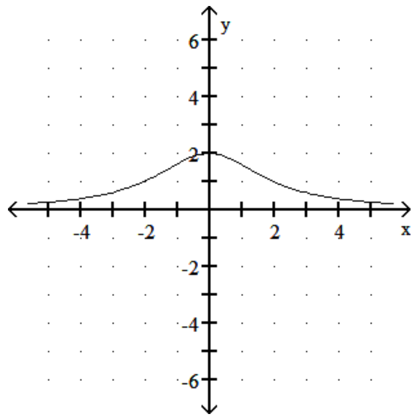
12) $y = \dfrac{8x}{x^2 + 4}$ 12) ______



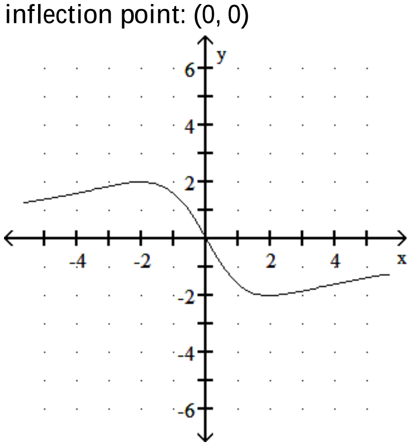
A) local minimum: (-2, -2)
local maximum: (2, 2)
inflection points: (0, 0), $(-2\sqrt{3}, -2\sqrt{3})$, $(2\sqrt{3}, 2\sqrt{3})$



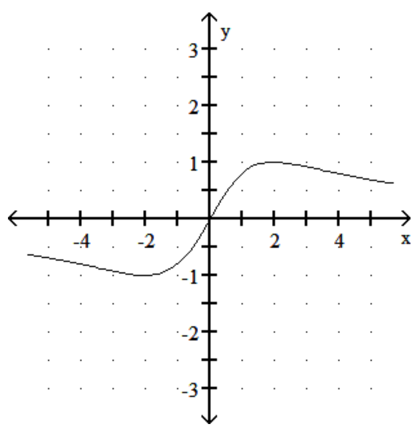
B) absolute maximum: (0, 2)
no inflection point



C) local minimum: (2, -2)
local maximum: (-2, 2)
inflection point: (0, 0)

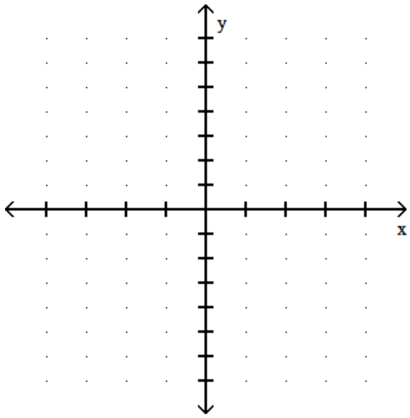


D) local minimum: (-2, - 1)
local maximum: (2, 1)
inflection point: (0,0)

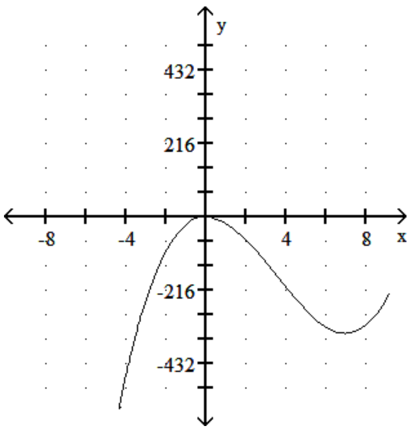


13) Graph the equation. Include the coordinates of any local extreme points and inflection points. 13) ______

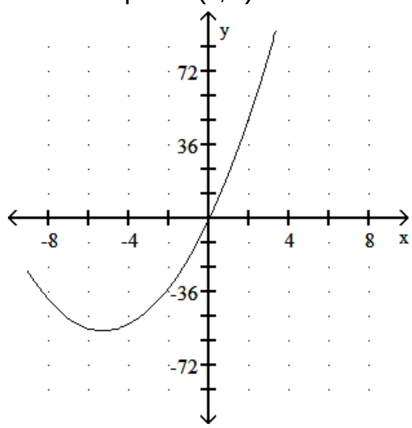
$y = 2x^3 + 12x^2 + 18x$



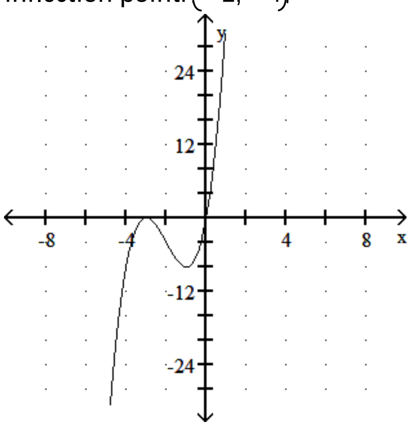
A) local minimum: (-7, 343)
local maximum: (0, 0)
inflection point: (-3.5, 171.5)



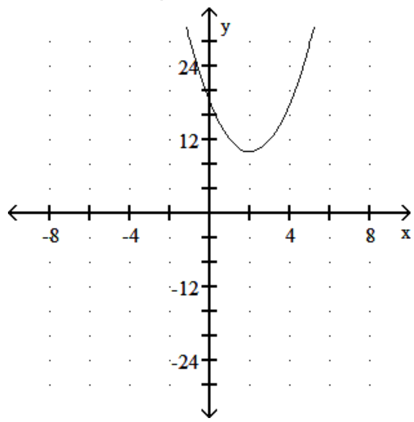
B) no extrema
inflection point: (0, 0)



C) local minimum: (-1, -8)
local maximum: (-3, 0)
inflection point: (- 2, - 4)

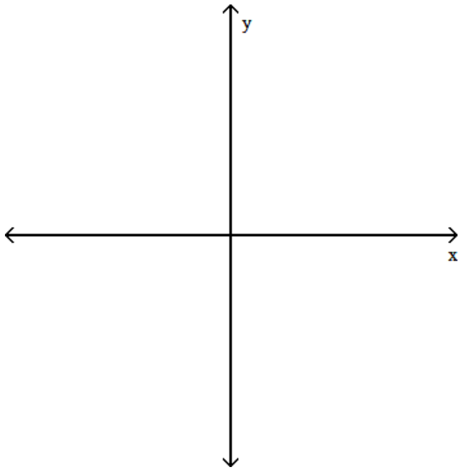


D) local minimum: (2, 10)
no inflection point

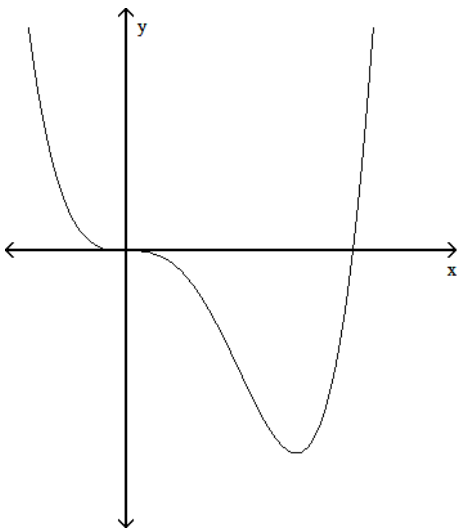


14) For the given expression y′, find y'' and sketch the general shape of the graph of y = f(x). 14) _______

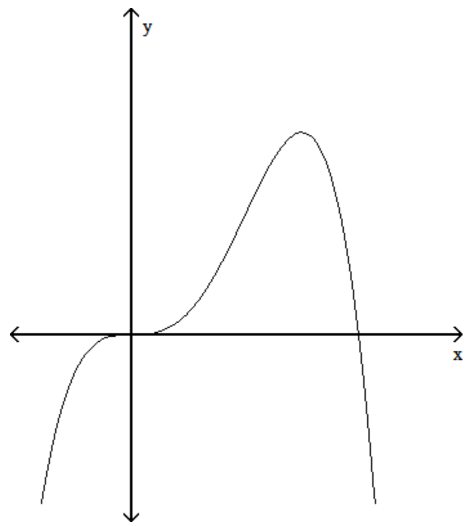
$y' = x^2(4 - x)$



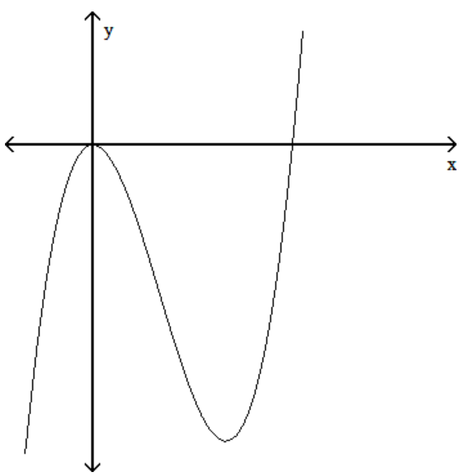
A)



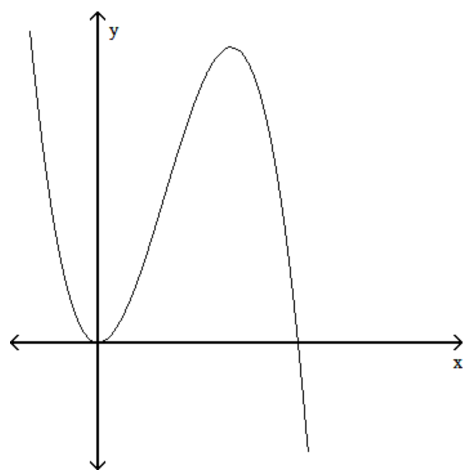
B)



C)



D)



Find the area enclosed by the given curves.

15) $y = x^3$, $y = 4x$ 15) ______

A) 8 B) 4 C) 16 D) 2

16) $y = \frac{1}{2}x^2$, $y = -x^2 + 6$ 16) ______

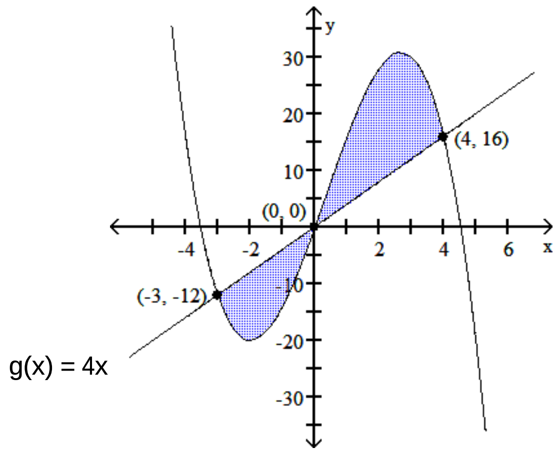
A) 16 B) 32 C) 8 D) 4

17) $y = \csc^2 x$, $y = \cot^2 x$, $x = \frac{\pi}{4}$, and $x = \frac{3\pi}{4}$ 17) ______

A) $\frac{3\pi}{4}$ B) π C) $\frac{\pi}{2}$ D) $\frac{\pi}{4}$

Find the area of the shaded region.

18) $f(x) = -x^3 + x^2 + 16x$ 18) ______



A) $\frac{1153}{12}$ B) $\frac{937}{12}$ C) $\frac{343}{12}$ D) $-\frac{343}{12}$

Find the volume of the solid generated by revolving the region bounded by the given lines and curves about the x-axis.

19) $y = \sqrt{\sin 4x}$, $y = 0$, $0 \le x \le \frac{\pi}{4}$ 19) ______

A) $\frac{1}{2}\pi$ B) 4π C) 2π D) 8π

20) $y = \sqrt{49 - x^2}$, $y = 0$, $x = 0$, $x = 7$ 20) ______

A) $\frac{1372}{3}\pi$ B) 14π C) 196π D) $\frac{686}{3}\pi$

21) $y = x^2 + 1$, $y = 3x + 1$ 21) ______

A) $\frac{207}{5}\pi$ B) 27π C) $\frac{63}{2}\pi$ D) $\frac{333}{5}\pi$

Find the volume of the solid generated by revolving the region about the y-axis.

22) The region enclosed by $x = \frac{y^2}{4}$, $x = 0$, $y = -4$, $y = 4$ 22) _______

A) $\frac{2048}{5}\pi$ B) $\frac{64}{5}\pi$ C) $\frac{32}{3}\pi$ D) $\frac{128}{5}\pi$

23) The region enclosed by the triangle with vertices (0, 0), (5, 0), (5, 2) 23) _______

A) $\frac{25}{3}\pi$ B) $\frac{100}{3}\pi$ C) $\frac{50}{3}\pi$ D) 100π

Find the volume of the solid generated by revolving the region about the given axis. Use the shell or washer method.

24) The region in the first quadrant bounded by $x = 5y - y^2$ and the y-axis about the y-axis 24) _______

A) $\frac{625}{3}\pi$ B) $\frac{625}{8}\pi$ C) $\frac{625}{6}\pi$ D) $\frac{625}{2}\pi$

25) The region bounded by $y = 4\sqrt{x}$, $y = 4$, and $x = 0$ about the line $y = 4$ 25) _______

A) $\frac{8}{3}\pi$ B) 8π C) $\frac{16}{3}\pi$ D) $\frac{4}{3}\pi$