MATH 1190.00 (Business Cal) **Exam-3** **Full Marks: 100**

Instructor: - Koshal Dahal **Date: 4/20/2011**

Each questions carry equal points:

Q.N.1: How old is a Chinese artifact that has lost 60% of its carbon-14?

Hint: $P(t) = p_0 e^{-kt}$, k= 0.0001205

present amount of Carbon-14 is 40 % So,

$$0.40\,P_0 = P_0\,e^{-0.0001205\,t}$$

$$0.40 = e^{-0.0001205\,t}$$

$$\ln 0.40 = -0.0001205\,t$$

$$\frac{\ln 0.40}{-0.0001205} = t$$

$$\therefore t \simeq 7604 \text{ years old.}$$

#

Q.N. 2: A home owner wants to have \$15,000 available in 5 years to pay for new siding. Interest is 6.1%, compounded continuously. How much money should be invested?

Hint: $P(t) = P_0 e^{rt}$

$$15{,}000 = P_0 e^{0.061(5)} = P_0 e^{0.305}$$

$$\frac{15000}{e^{0.305}} = P_0 \implies P_0 \approx \underline{11{,}05\cancel{7}}$$

#

Q.N.3: find the derivative of the following functions w.r.to x,

a) $f(x) = 7^x (\log_4 x)^9$ b) $f(x) = 3^{x-1}$

(a) By Product rule

$$f'(x) = 7^x \cdot \left[9 \left(\log_4 x\right)^8 \cdot \frac{1}{x} \cdot \frac{1}{\ln 4} \right] + \left(\log_4 x\right)^9 \cdot 7^x \ln 7 .$$

$$= \frac{9 \cdot 7^x \left(\log_4 x\right)^8}{x \cdot \ln 4} + 7^x \cdot \ln 7 \cdot \left(\log_4 x\right)^9$$

#

(b) P.T.O.

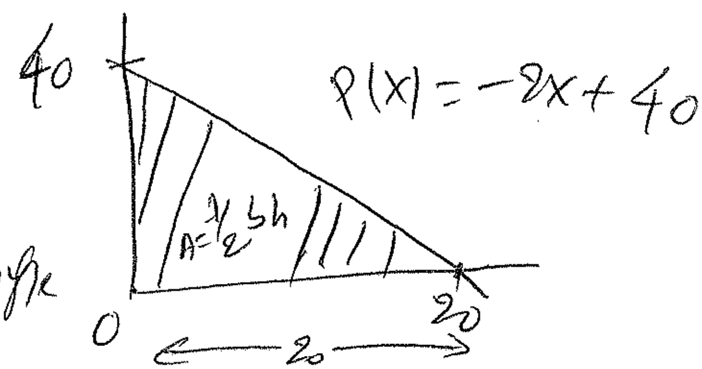
ans (3) $f'(x) = 3^{x-1} \cdot \ln(3)$. #

Q.N.4: An Apple company has a marginal profit function P(x) = -2x+40, where P(x) is in dollars per unit. Then find the total profit from the productions and sale of the first 20 units.

Hint: use area of trapezoid or triangle

$h = P(0) = 40$

$b = 20$



Total Cost = area of triangle

$= \frac{1}{2} bh$

$= \frac{1}{2} \times 40 \times 20$

$= \$400$. #

Q.N.5: Find the anti-derivative of:

a) $\frac{5}{x^2} + \frac{3}{x}$

$\int \left(\frac{5}{x^2} + \frac{3}{x} \right) dx$

$= \int 5 \cdot x^{-2} dx + 3 \int \frac{1}{x} dx$

$= \frac{5 x^{-2+1}}{-2+1} + 3 \cdot \ln x + C$

$= -\frac{5}{x} + 3 \ln x + C.$

b) $2e^{5x}$

$\int 2e^{5x} dx$

$= 2 \frac{e^{5x}}{5} + C$

#

Q.N.6: Find the area of given function over the given interval

(a) $Y = 2-x^2-x$; [-2, 1]

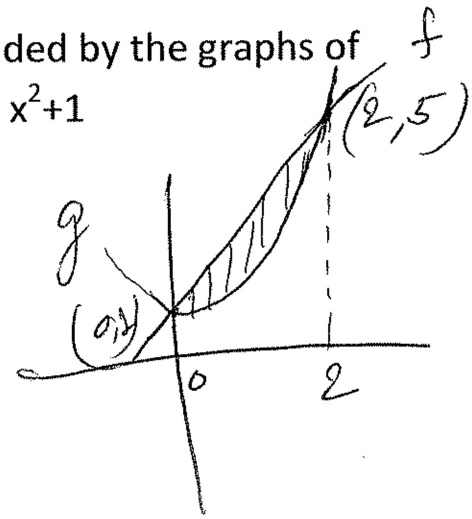
$$\int_{-2}^{1} y\,dx = \int_{-2}^{1}(2-x^2-x)\,dx = \left[2x-\frac{x^3}{3}-\frac{x^2}{2}\right]_{x=-2}^{1}$$

$$= \left(2-\frac{1}{3}-\frac{1}{2}\right)-\left(-4+\frac{8}{3}-\frac{4}{2}\right)$$

$$= 2-\tfrac{1}{3}-\tfrac{1}{2}+4-\tfrac{8}{3}+\tfrac{4}{2}$$

$$= \tfrac{9}{2}$$

(b) Find the area of the region that is bounded by the graphs of following functions: $f(x)= 2x+1$ and $g(x)= x^2+1$



Point of intersection

$$2x+1 = x^2+1$$

$$0 = x^2+1-2x-1$$

$$= x^2-2x$$

$$= x(x-2)$$

Either $x=0$, or $x=2$.

$\therefore$ As in fig, req^d Area $= \int_0^2 f(x)\,dx - \int_0^2 g(x)\,dx$

$$= \int_0^2 (2x-x^2)\,dx = \left[x^2-\frac{x^3}{3}\right]_0^2$$

$$= 4-\tfrac{8}{3} = \tfrac{4}{3} \quad \#$$

Q.N.7: Evaluate by <u>substitution</u>:

(a) $\int xe^{x^2}\,dx$

Put $x^2 = u$

$2x\,dx = du$ (by diff.)

$x\,dx = du/2$.

$\therefore$ given integral

$= \int e^u \cdot \frac{du}{2}$

$= \frac{1}{2} \cdot e^u + C$

$= \frac{1}{2} e^{x^2} + C$ //

(b) $\int 3x^2(x^3+1)^5\,dx$

Put $x^3+1 = u$

$3x^2\,dx = du.$ (diff.)

$\therefore = \int u^5 \cdot du = \frac{u^6}{6} + C$

$= \frac{(x^3+1)^6}{6} + C$ //

Q.N.8: Evaluate using <u>integration by parts</u>

a) $\int x^3 \ln(2x)\, dx$

$u = \ln(2x)$ | $dv = x^3 dx$

$du = \frac{\cancel{1}}{\cancel{2}x} \cdot \cancel{2} = \frac{1}{x} \cdot dx$ | $v = \frac{x^4}{4}$

By integration by parts

$= uv - \int v\, du$

$= \ln(2x) \cdot \frac{x^4}{4} - \int \frac{x^4}{4} \cdot \frac{1}{x}\, dx$

$=$ " $- \frac{1}{4} \int x^3\, dx$

$=$ " $- \frac{x^4}{16} + C$ #

b) $\int (x^3+4)(3x^2)dx$

Put $x^3+4 = u$ | $dv = 3x^2 dx$

$du = 3x^2 dx$ | $v = x^3$

By inte. by parts

$= (x^3+4)x^3 - \int x^3 \cdot 3x^2 dx$

$=$ " $- 3\int x^5 dx$

$=$ " $- \frac{3x^6}{6} + C$

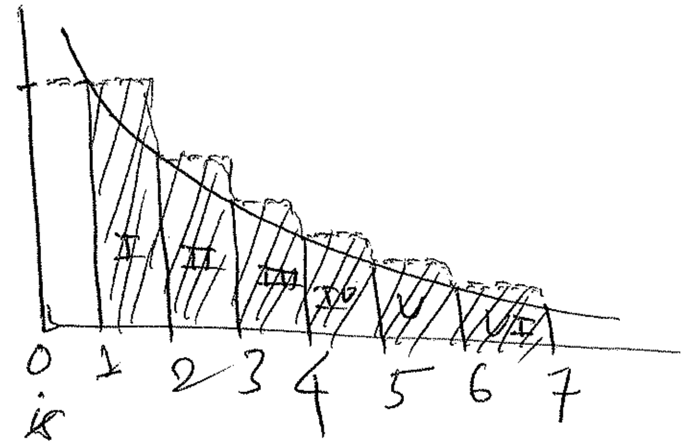
$= x^3(x^3+4) - \frac{x^6}{2} + C$

#

Bonus: [10]

Approximate the area under the graph of $f(x) = 1/x^2$ over the interval [1,7] by computing the area of each rectangle to four decimal places and then adding.

$\Delta x = \frac{b-a}{n} = \frac{7-1}{6} = 1.$ $f(1)$



~~[illegible]~~

The area of rectangle I is

$A_1 = f(1)\cdot\Delta x = \frac{1}{1^2} \times 1 = 1$

Likewise,

$A_2 = f(2)\cdot\Delta x = \frac{1}{4} \times 1 = 0.2500$

$A_3 = f(3)\cdot\Delta x = \frac{1}{9} \times 1 = 0.1111$

$A_4 = f(4)\cdot\Delta x = \frac{1}{16} \times 1 = 0.0625$

$A_5 = f(5)\cdot\Delta x = \frac{1}{25} \times 1 = 0.0400$

$A_6 = f(6)\cdot\Delta x = \frac{1}{36} \times 1 = 0.0278$

Adding all areas gives us req^d area of $\simeq 1.4914$.

//

The End!!!