Research Statement

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My research is primarily concerned with the closely related areas of set theory: *inner models for large cardinals*, *determinacy*, and *descriptive set theory*.

One area of research in which I am active involves producing inner models of large cardinals from combinatorial properties of small cardinals through a technique known as the *core model induction* discovered by H. Woodin (Berkeley) and first used to show that the existence of an ω_1-dense ideal on ω_1 implies that the *Axiom of Determinacy* holds in the inner model $L(\mathbb{R})$. Earlier Woodin had proved that $\mathrm{AD}^{L(\mathbb{R})}$ implies the existence of an inner model model with ω Woodin cardinals. The general picture is:

combinatorial principle at a small cardinal
$\Longrightarrow$ the existence of an inner model for the axiom of determinacy
$\Longrightarrow$ the existence of inner models of large cardinals

The "inner models of determinacy $\Longrightarrow$ inner models of large cardinals" part has been known for a while and without going into details this part can be strengthened to

stronger inner models of determinacy
$\Longrightarrow$ inner models for *stronger* large cardinals

In particular Woodin's *Mouse Set Theorem* shows that if there exists a set of reals A such that $L(A,\mathbb{R})$ is a model of determinacy and moreover A is not definable in $L(A,\mathbb{R})$ from real and ordinal parameters, then there is an inner model falling just short of "*there are cardinals $\kappa < \delta$ such that δ is Woodin and κ is strong past δ.*" A more technically correct statement of this fact is

$$\exists A \subseteq \mathbb{R}\, L(A,\mathbb{R}) \models \text{“}\mathrm{AD}^+ + \Theta_0 < \Theta\text{”} \Longrightarrow \exists \mathcal{M}\,\big(\mathcal{M} \textit{ is a non-tame mouse}\big)$$

I managed to show assuming Cantor's *Continuum Hypothesis* together with the existence of an even nicer ideal on ω_1 that such a set of reals exists, thus producing an instance of

stronger combinatorial principles at a small cardinals
$\Longrightarrow$ the existence of *stronger* inner models of determinacy

In fact the technique shows quite a bit more, actually obtaining a model of $\mathrm{AD}_{\mathbb{R}}$ from the same combinatorial hypothesis. The general question of obtaining strong models of determinacy from combinatorial principles is wide open. A nice prospect is the *proper forcing axiom* (PFA). Recently, J. Steel (Berkeley) has shown PFA $\Longrightarrow \mathrm{AD}^{L(\mathbb{R})}$. It is expected that PFA is quite strong and we should be able to show PFA implies the existence of non-tame mice and much more.

An important ingredient in obtaining the model $L(A,\mathbb{R})$ mentioned above is a calculation of the inner model HOD in (certain) models of determinacy as a limit of a directed system of mice together with some additional information on how to *iterate* the limit. This calculation of HOD, which is due to Woodin, forms the base for a new kind of inner model theory that is still in its infancy, but which already has far reaching implications. Steel first used a closely related directed system of mice to calculate a fragment of HOD as a mouse and used this to answer descriptive set theoretic questions whose solutions were unattainable using previously developed techniques of descriptive set theory. For example, Steel showed that all regular cardinals of $L(\mathbb{R})$ are measurable and that the *Generalized Continuum Hypothesis* holds in HOD. Steel's work also showed that certain inner models that arise in naturally in descriptive set theory can be viewed as direct limits of appropriate directed systems of mice in the same way HOD is. This suggests that these directed systems of mice can be used to answer further descriptive set theoretic questions left open.

Lately S. Jackson (UNT) and I have been working on utilizing directed systems of mice to generalize some descriptive set theoretic results. In particular we are working on generalizing a theorem of A. Kechris (CalTech) and D. Martin (UCLA) on the closure of certain pointclasses under bounded ordinal quantification. We are also attempting to use these systems to show that every regular Suslin cardinal satisfies the strong partition property. The ultimate goal of this research is to push (some form of) Jackson's fine analysis of the structure of the projective cardinals through all of the regular cardinals of $L(\mathbb{R})$. The calculation of the direct limit of the relevant system involves calculating the simplest iteration strategies required for comparing all of the mice in that system; this in turn is an interesting question in the realm of inner model theory.

An itroduction to the ways in which inner models for large cardinals, descriptive set theory, and determinacy are interrelated can be found on my website at `http://www.math.unt.edu/~ketchers`.