Math 4050 Practice Problem Set #10

At the top of your write-up, you must also write a statement attesting that you have at least thought about all assigned problems. Points will be deducted if you do not write this statement. This does not mean that you solved all of the problems — just that you gave some thought about how to solve every problem. For the sake of preparing for the state certification exam, as well as for your own integrity, I'd prefer that you are honest when writing this statement.

Problem 10.1 Find the equation of the tangent line to $y = \sqrt{x^2 + 9}$ at $x = 4$. Write your answer in slope-intercept form.

Problem 10.2 Differentiate

$$f(\theta) = \frac{\theta \sin^2 \theta^2}{1 + \sec 2\theta}$$

Problem 10.3 State the formal definition of a derivative. Use this formal definition to find the derivative of $f(x) = 5x - 3$. You *must* use the formal definition of a derivative to do this problem.

Problem 10.4 If possible, sketch the graph of a function so that f is continuous at $x = 2$ but $f'(2)$ does not exist. If this cannot be done, write the word "Impossible."

Problem 10.5 If possible, sketch the graph of a function so that $f'(2) = 0$ but f is discontinuous at $x = 2$. If this cannot be done, write the word "Impossible."

Problem 10.6 True or false:

- If $f(x)$ is continuous at a point c, then f must be differentiable at c.

- If $f(x)$ is differentiable at a point c, then f must be continuous at c.

Problem 10.7 If possible, sketch a function f which satisfy all of the following:

- $\lim\limits_{x \to 4^+} f(x) = 3$

- $\lim\limits_{x \to 4^-} f(x) = -2$

- $f(4) = 1$

- f is differentiable for all $x \neq 4$

If this is not possible, write the word "Impossible."

Problem 10.8 If possible, sketch a function f which satisfy all of the following:

- $\lim\limits_{x \to \infty} f(x) = 3$

- $\lim\limits_{x \to -\infty} f(x)$ is undefined but is neither ∞ or $-\infty$.

- f is differentiable for all x

If this is not possible, write the word "Impossible."

Problem 10.9 Find $\dfrac{dy}{dx}$ if

$$\sqrt{x^3 + y^2} = \cot\left(\frac{x}{y}\right).$$

Problem 10.10 Differentiate $y = \csc^2\left(x + \sqrt[3]{2x + 7}\right)$.

Problem 10.11 Find the values of x where $Q(x) = \dfrac{x}{x^2 + 1}$ has a horizontal tangent line.

Problem 10.12 Find $\dfrac{d^2 y}{dx^2}$ if $y = \dfrac{1}{x^2 + 1}$.

Problem 10.13 Differentiate $y = x^2 \sin^6\left(\sec(x \tan 4x)\right)$.

Problem 10.14 Differentiate

$$h(t) = \frac{\sqrt{t}}{4} - \frac{\sqrt[3]{t}}{5} + \frac{\sqrt[6]{t}}{10}.$$

Problem 10.15 Find the values of x where the slope of the tangent line to

$$f(x) = \frac{x}{2x + 1}$$

has slope 1.