Math 4050 **Practice Problem Set #8**

At the top of your write-up, you must also write a statement attesting that you have at least thought about all assigned problems. Points will be deducted if you do not write this statement. This does not mean that you solved all of the problems — just that you gave some thought about how to solve every problem. For the sake of preparing for the state certification exam, as well as for your own integrity, I'd prefer that you are honest when writing this statement.

Problem 8.1 Find the coefficient of the x^9−term in the expansion of $(2x^3 - 1)^{12}$. *You do not need to give the entire expansion to receive full credit.*

Problem 8.2 Compute $\sum_{k=1}^{\infty} 2\left(-\frac{2}{5}\right)^{2k-1}$.

Problem 8.3 Express in $\sum$−notation:

$$q^2 - pq^6 + \frac{p^2q^{10}}{2} - \frac{p^3q^{14}}{6} + \frac{p^4q^{18}}{24} - \frac{p^5q^{22}}{120}$$

Problem 8.4 Use mathematical induction to show that

$$2 + 2^2 + 2^3 + \ldots + 2^n = 2^{n+1} - 2$$

Problem 8.5 For an arithmetic sequence, $a_4 = 35$ and $a_8 = 75$. Find the sum of the first 100 terms.

Problem 8.6 Consider the sequence recursively defined by

$$a_n = \begin{cases} 1, & n = 1 \\ a_{n-1} + 1 + 2\sqrt{a_{n-1}}, & n \geq 2 \end{cases}$$

- Find a_1, a_2, a_3, and a_4.
- Now guess a formula for a_n, and use mathematical induction to prove that your formula works.

Problem 8.7 Expand and simplify $(1 - \sqrt{2})^6$.

Problem 8.8 Use the formula

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}.$$

to calculate

$$\sum_{k=1}^{12} \left(\frac{k^2}{5} + 2\right)$$

Problem 8.9 Evaluate the arithmethic series

$$0.9 + 1.3 + 1.7 + \ldots + 8.1 + 8.5$$

Problem 8.10 Use mathematical induction to show that $2^{2n-1} + 1$ is always divisible by 3.

Problem 8.11 Use mathematical induction to show that

$$1 + 3 + 5 + \ldots + (2n - 1) = n^2$$

Problem 8.12 Compute

$$\sum_{k=3}^{100}(2k-3)$$

Problem 8.13 An infinite geometric sequence has −1000 and 8 as its second and fifth terms, respectively. Find the sum of this infinite geometric series.

Problem 8.14 Use the principle of telescoping series to exactly evaluate

$$\sum_{n=1}^{99}\left([n+1]^4 - n^4\right)$$

Problem 8.15 Use mathematical induction to show that

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\ldots+\frac{1}{n(n+1)}=\frac{n}{n+1}.$$