Math 4050 **Practice Problem Set #6**

Revised

At the top of your write-up, you must also write a statement attesting that you have at least thought about all assigned problems. Points will be deducted if you do not write this statement. This does not mean that you solved all of the problems — just that you gave some thought about how to solve every problem. For the sake of preparing for the state certification exam, as well as for your own integrity, I'd prefer that you are honest when writing this statement.

Problem 6.1 Prove the trigonometric identity

$$8\sin^2\theta\cos^2\theta = 1 - \cos 4\theta$$

Problem 6.2 Simplify

$$\cos\left(\cos^{-1}x - \tan^{-1}\left(\frac{2}{3}\right)\right)$$

Problem 6.3 Prove the trigonometric identity

$$\frac{\sin x}{\cot x} = \sec x - \cos x$$

Problem 6.4 Use a calculator to compute

$$\sin 20^\circ \sin 40^\circ \sin 80^\circ.$$

Then prove this result using trigonometric identities.

Problem 6.5 Find the exact value of

$$\sin\left(\frac{17\pi}{24}\right)\sin\left(\frac{\pi}{24}\right)$$

Problem 6.6 Use the substitution $u = 3\tan\theta$ to simplify

$$\frac{6u^2}{(9+u^2)^{3/2}}$$

Your final answer should contain no fractions.

Problem 6.7 Prove the trigonometric identity

$$\frac{\cos 3x - \cos 7x}{\sin 3x + \sin 7x} = \frac{2\tan x}{1 - \tan^2 x}$$

Problem 6.8 Suppose that $0 < x < 2\pi$, $\sin x = -\frac{5}{13}$, and $\cos x > 0$. Find $\cos\left(\frac{x}{2}\right)$ exactly.

Problem 6.9 Prove the trigonometric identity

$$\sin 3x = (1 + 2\cos 2x)\sin x$$

Problem 6.10 Suppose that $\pi < x < 2\pi$ and $\cos x = \frac{7}{25}$. Use half-angle identities twice to find $\sin\left(\frac{x}{4}\right)$ exactly.

Problem 6.11 Prove the trigonometric identity

$$(1 - \cos 2x)(1 + \cos 2x) = \frac{1}{\csc^2(2x)}$$

Problem 6.12 Exactly evaluate

$$\cos\left(2\tan^{-1}(-3)\right)$$

Problem 6.13 Determine k and ϕ so that

$$-2\sin 2x + 3\cos 2x = k\sin(2x + \phi).$$

Express ϕ in degrees, accurate to one decimal place.

Problem 6.14 Solve the trigonometric equation over all real numbers:

$$2\cos^2(2x) - 11\cos(2x) + 5 = 0$$

Problem 6.15 Solve the equation

$$\tan 2x = -\sqrt{3}$$

Problem 6.16 Find all solutions of

$$\sin x - \sqrt{3}\cos x = -1.$$

Hint: Begin by converting the left-hand side into a single trigonometric function.

Problem 6.17 Solve for x on the interval $[0, 2\pi]$:

$$\sin 2x + \sin 4x = 0$$

Hint: Use a trig identity to get started.

Problem 6.18 Solve the trigonometric equation over all real numbers. Express your answer in a form that you would plug into a calculator.

$$\sin 3x = \frac{1}{3}$$

Problem 6.19 Simplify

$$f(x) = 2\sin^{-1}\left(\frac{\sqrt{x}}{2}\right) - \sin^{-1}\left(\frac{x-2}{2}\right).$$

Hints:

- Use a graphing calculator to plot $y = f(x)$. You should be able to guess the answer.
- Confirm your answer by computing $\sin[f(x)]$.