Math 4050 **Practice Problem Set #5**

At the top of your write-up, you must also write a statement attesting that you have at least thought about all assigned problems. Points will be deducted if you do not write this statement. This does not mean that you solved all of the problems — just that you gave some thought about how to solve every problem. For the sake of preparing for the state certification exam, as well as for your own integrity, I'd prefer that you are honest when writing this statement.

Problem 5.1 Simplify $\sin\left[\cos^{-1}\left(-\frac{5}{9}\right)\right]$.

Problem 5.2 Simplify the expression

$$\frac{\tan x}{1+\sec x} - \frac{\tan x}{1-\sec x}$$

Your final answer should *not* include any fractions.

Problem 5.3 Change the equation

$$r = \frac{4}{3 - 3\sin\theta}$$

into an equation in rectangular coordinates. Your final answer should have only y on one side of the equation and an expression involving x on the other side.

Problem 5.4 Evaluate the given expressions. No partial credit will be given for incorrect answers.

- $\tan^{-1}(1) =$ ____________
- $\sin^{-1}\left(\frac{1}{2}\right) =$ ____________
- $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) =$ ____________
- $\cos^{-1}(0) =$ ____________
- $\sin^{-1}(-1) =$ ____________
- $\tan^{-1}\left(\sqrt{3}\right) =$ ____________
- $\cos^{-1}\left(-\frac{1}{2}\right) =$ ____________
- $\tan^{-1}\left(\tan\frac{3\pi}{4}\right) =$ ____________
- $\sin\left(\sin^{-1}(0.7)\right) =$ ____________
- $\cos^{-1}\left(\cos\frac{7\pi}{6}\right) =$ ____________

Problem 5.5 As we breathe, our lungs decrease and increase in volume. The volume of air that we inhale and exhale with each breath is called the *tidal volume*.

Suppose a man watching television breathes once every 5 seconds. His average lung capacity is 2500 mL, and his tidal volume is 500 mL. Assuming that his lung capacity oscillates in simple harmonic motion, find a formula for $V(t)$, the volume of the lungs after t seconds. You may assume that the lungs are at their maximum volume at time 0.

Problem 5.6 Simplify the expression $\cos\left[\tan^{-1}(2x)\right]$.

Problem 5.7 In rectangular coordinates, the point P is represented by $(-4, 4)$. Find three different representations of P using polar coordinates.

Problem 5.8 Sketch two cycles of the graph of $y = 2\sin\left(2x - \dfrac{\pi}{2}\right) + 3$.

Problem 5.9 Use the substitution $u = 3\sec x$ to simplify $\dfrac{u^4}{(u^2-9)^2}$.

Problem 5.10 Simplify the expression $\sin x + \cos x \cot x$.

Problem 5.11 Sketch the graph of

$$r = \frac{3}{1+0.5\sin\theta} = \frac{6}{2+\sin\theta}$$

Hint: You can draw an acceptable sketch after plotting only four points.

Problem 5.12 Find numbers a, b, ϕ and c so that the graph below may be represented as

$$f(x) = a\sin(b[x-\phi]) + c$$

Problem 5.13 Find the angles in $\triangle ABC$ if $a = 12$, $b = 14$, and $c = 15$.

Problem 5.14 Triangle $\triangle ABC$ has sides $a = 9$, $b = 10$, and $c = 15$. Find the area of $\triangle ABC$, rounded to two decimal places.

Problem 5.15 Triangle $\triangle ABC$ has sides $a = 15$, $b = 19$, and $\alpha = 40°$. Find c, accurate to one decimal place.

Problem 5.16

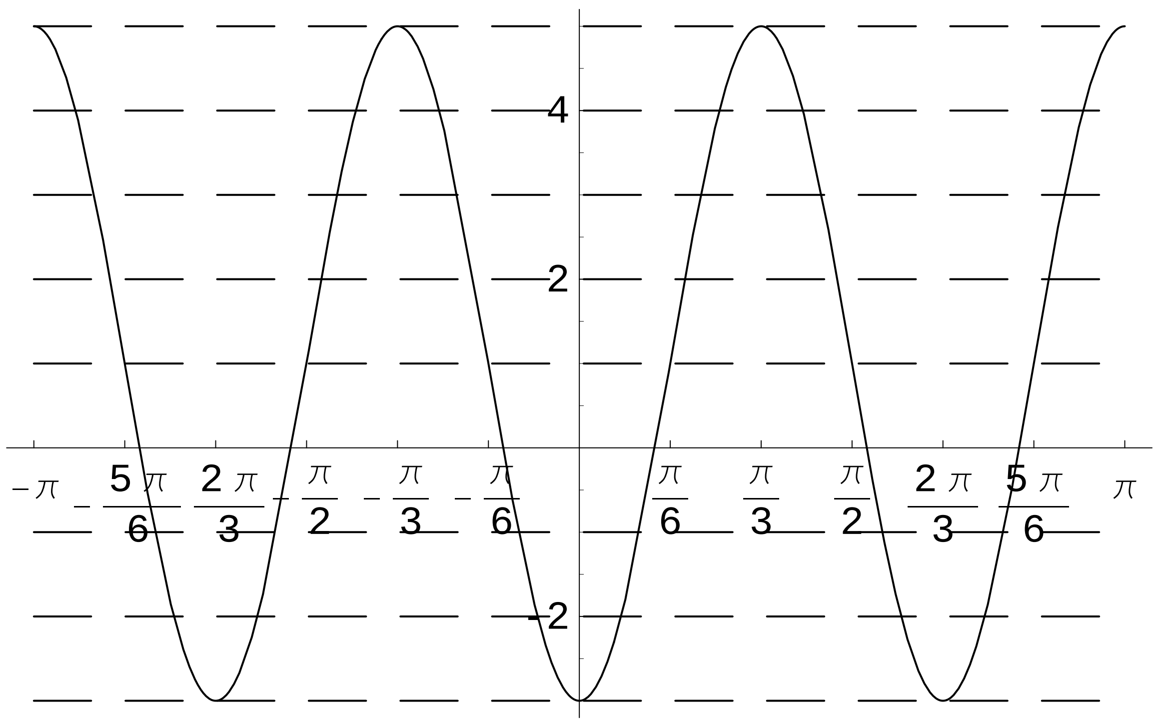
1. Find the set of all x so that $\sin^{-1}(\sin x) = x$. Express your answer in interval notation.
2. Find the set of all x so that $\sin\left(\sin^{-1} x\right) = x$. Express your answer in interval notation.

Problem 5.17

1. Find the set of all x so that $\cos^{-1}(\cos x) = x$. Express your answer in interval notation.
2. Find the set of all x so that $\cos\left(\cos^{-1} x\right) = x$. Express your answer in interval notation.

Problem 5.18

1. Find the set of all x so that $\tan^{-1}(\tan x) = x$. Express your answer in interval notation.
2. Find the set of all x so that $\tan\left(\tan^{-1} x\right) = x$. Express your answer in interval notation.



4
2
-2
$-\pi$
$-\frac{5\pi}{6}$
$\frac{2\pi}{3}$
$-\frac{\pi}{2}$
$-\frac{\pi}{3}$
$-\frac{\pi}{6}$
$\frac{\pi}{6}$
$\frac{\pi}{3}$
$\frac{\pi}{2}$
$\frac{2\pi}{3}$
$\frac{5\pi}{6}$
π