Review Problem 1.1 Three dice are rolled. The first has 2 red faces and 4 green faces. The second has 3 red faces and 3 green faces. The third has 4 red faces and 2 green faces.

- What is the chance of getting 1 red and 2 greens?
- Given 1 red and 2 greens, what is the probability that the third die came up red?

Review Problem 1.2 A cereal company puts a prize in 95% of their boxes. If you buy 500 boxes, estimate the chance that you will collect at least 480 prizes.

Review Problem 1.3 Suppose $P(E \mid F) = 0.2$, $P(F \mid E) = 0.4$, and $P(E \cap F) = 0.1$.

- Find $P(F)$.
- Find $P(E^c F)$.
- Find $P(E^c \mid F^c)$.

Review Problem 1.4 The fraction of persons in a population who have a certain disease is 0.01. A diagnostic test is available to test for the disease. But for a healthy person, the chance of being falsely diagnosed is 0.05, while for someone with the disease the chance of being falsely diagnosed as healthy is 0.2. Suppose the test is performed on someone chosen at random.

- What are the sensitivity and specificity?
- What is the probability that the test is positive?
- What is the probability that the test is negative and he has the disease?
- What is the probability that the test is negative?
- Suppose the test comes back positive. What is the probability that he has the disease?

Review Problem 1.5 A point (x, y) is chosen at random from the unit square

$$\{(x, y) : 0 \le x, y \le 1\}.$$

What is the probability that $x + y \le 1$ given that $x \le 1/2$?

Review Problem 1.6 Three independent events have probabilities 0.1, 0.2 and 0.3. What is the probability that

- None occur
- At least one occurs
- Exactly one occurs

Review Problem 1.7 If 1000 raffle tickets are sold, of which 50 are winning tickets, and you purchase 10, what is probability that you will have 2 winning tickets?

Review Problem 1.8 In a group of four people, what is the probability that no two will have the same birth month? (Assume that being born in each month is equally likely.)

Review Problem 1.9 In a fish-tagging survey, 100 bass are netted, tagged, and released. Some time later, after the bass have had a chance to disperse, 100 more bass are caught. In this second sample, 5 of the 100 are found to have tags. If the total number of bass in the lake is n, what is the probability of this happening? Your answer will depend on n.

(I'm not asking you to do the following. But, for what it's worth, statisticians would take the next step and find the value of n that maximizes the probability that you found above. This value of n would serve as an estimate for the number of bass in the lake.)

Review Problem 1.10 A bowl contains w white chips, r red chips and b black chips. Chips are drawn with replacement. What is the probability that a white chip will appear before a black chip?

Review Problem 1.11 Two cards are dealt from a well-shuffled deck. Find the probability that the first is a spade or the second is a heart.

Review Problem 1.12 Prove that

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$

Review Problem 1.13 A bus tour operator uses a bus with a capacity of 90 people but sells 100 tickets. Historically, the operator knows that one of every 12 tickets sold is a no-show. What is the probability that everyone who shows up is accommodated?

Review Problem 1.14 Let X and Y be independent random variables uniformly distributed on $\{1, 2, 3, \ldots, n\}$. Calculate $P(X = Y)$ and $P(X \geq Y)$.

Review Problem 1.15 The number of phone calls per minute that arrives into phone bank follows a Poisson(3) distribution. Find the probability that, in the next two minutes, at least 3 calls arrive.

Review Problem 1.16 You go to a beach party. Three of you are bringing coolers with sandwiches. Your cooler has 10 ham sandwiches and 5 cheese sandwiches. The other two coolers each have 3 ham sandwiches and 17 cheese sandwiches. Someone takes a sandwich at random from one of the coolers. What is the probability that the sandwich is a cheese sandwich?

Review Problem 1.17 Six cards are dealt without replacement from an ordinary deck of cards. Find the probability that two of the first five cards are spades and the sixth card is a spade.

Review Problem 1.18 A young boy has five blue and four white marbles in his left pocket. In his right pocket, he has four blue and five white marbles. He pulls one marble at random from his left pocket and places it in his right pocket. Then he draws a marble from his right pocket. What is the probability that second drawn marble is blue?

Review Problem 1.19 Suppose that A and B are independent events. Prove that A and B^c are also independent.

Review Problem 1.20 A bin contains 25 light bulbs.

- Five light bulbs are in good condition and will work for at least 30 days.
- Ten light bulbs are partially defective and will fail in their second day of use.
- Ten light bulbs are totally defective and will not light up.

Given that a randomly chosen bulb initially lights, what is the probability that it will still be working after one week?

Review Problem 1.21 A multiple-choice test has m possible answers for each question. Let p be the probability that a student knows the answer to a question. The student will answer correctly if the answer is known but will guess from among the m alternatives if the answer is unknown. Given that a student answered a question correctly, what is the probability that the student knew the correct answer?

Your answer will involve both p and m. Then find the numerical answer if $m = 5$ and $p = 1/2$.

Review Problem 1.22 A communication system consists of n components, and each component will work with probability p independently of the other components. The system will work if at least one-half of the components work. Find the values for p for which a 5-component system will be more likely to work than a 3-component system.

Review Problem 1.23 Moe, Larry, and Curly each roll a die. Let A be the event that Moe and Larry roll the same number. Let B be the event that Moe and Curly roll the same number. Let C be the event that Larry and Curly roll the same number.

- Show that A, B, and C are pairwise independent.
- Show that A, B, and C are not independent.

Review Problem 1.24 Give a formula for $P(A \mid B^c)$ in terms of only $P(A)$, $P(B)$ and $P(A \cap B)$.

Review Problem 1.25 Suppose A and B are two events with $P(A) = 0.5$ and $P(A \cup B) = 0.8$.

- For what value of $P(B)$ would A and B be mutually exclusive?
- For what value of $P(B)$ would A and B be independent?