Math 4050 **Practice Problem Set #2**
(revised)

At the top of your write-up, you must also write a statement attesting that you have at least thought about all assigned problems. Points will be deducted if you do not write this statement. This does not mean that you solved all of the problems — just that you gave some thought about how to solve every problem. For the sake of preparing for the state certification exam, as well as for your own integrity, I'd prefer that you are honest when writing this statement.

Problem 2.1 Solve the system of equations:

$$\begin{aligned} 2x + y - 2z &= 7 \\ x + y + z &= -1 \\ -2y - z &= -3 \end{aligned}$$

Problem 2.2 Solve the system of equations:

$$\begin{aligned} 2x + y - 3z &= -3 \\ 3x - 2y + 4z &= 2 \\ 4x + 2y - 6z &= -7 \end{aligned}$$

Problem 2.3 Solve the system of equations:

$$\begin{aligned} 3x - 2y + z &= -1 \\ 2x + y - z &= 5 \\ 10x - 2y &= 8 \end{aligned}$$

Problem 2.4 Solve the system of equations, using complex numbers if necessary:

$$\begin{aligned} x^2 + y^2 &= 25 \\ 2x + 3y &= 19 \end{aligned}$$

Problem 2.5 Solve the system of equations, using complex numbers if necessary:

$$\begin{aligned} \frac{(x-2)^2}{9} - \frac{(y+3)^2}{16} &= 1 \\ x - 3y &= 24 \end{aligned}$$

Problem 2.6 Solve the system of equations, using complex numbers if necessary:

$$\begin{aligned} x^2 + 3xy + y^2 &= 20 \\ xy - y^2 &= 0 \end{aligned}$$

Problem 2.7 Graph in the plane the solutions for the following system of inequalities:

$$\begin{aligned} 2x + y &\geq 4 \\ x - y &< 2 \end{aligned}$$

Problem 2.8 Find the maximum value of the objective function $f(x, y) = 2x + y$ for all (x, y) that satisfy the following conditions:

$$\begin{aligned} x + y &\leq 4 \\ 3x + y &\leq 6 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

Problem 2.9 Find the minimum value of the objective function $f(x, y) = 36x + 40y$ for all (x, y) that satisfy the following conditions:

$$\begin{aligned} 3x + 2y &\geq 18 \\ 3x + 4y &\geq 24 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

Problem 2.10 Express the solution of this inequality in interval notation:

$$-2\left|3 - \frac{x}{5}\right| - 1 \geq -3$$

Problem 2.11 Express the solution of this inequality in interval notation:

$$3|7 - 5x| - 9 > 15$$

Problem 2.12 Suppose that

$$A = \begin{bmatrix} -2 & 3 & 1 \\ 5 & 2 & 4 \\ 2 & 0 & -1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -7 & 5 & -3 \\ 1 & 9 & 0 \\ 2 & -2 & -5 \end{bmatrix}.$$

Find the following quantities, if possible.

- $6B$
- $-A$
- $2A - 3B$
- A^2
- AB
- BA
- A^{-1}

Problem 2.13 Suppose that

$$A = \begin{bmatrix} -2 & 3 & 1 & 7 \\ 5 & 2 & 4 & 8 \end{bmatrix} \qquad \text{and} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ -5 & 2 & -1 \\ 2 & 8 & 5 \\ 4 & -3 & -1 \end{bmatrix}.$$

Find the following quantities, if possible.

- $6B$
- $-A$
- $2A - 3B$
- A^2
- AB
- BA
- A^{-1}

Problem 2.14 Calculate (if possible) the determinants of the following matrices.

- $A = \begin{bmatrix} 3 & 4 \\ 1 & -3 \end{bmatrix}$
- $B = \begin{bmatrix} -2 & 5 & 1 \\ 4 & 2 & 4 \\ 2 & 0 & -1 \end{bmatrix}$
- $C = \begin{bmatrix} -2 & 3 & 1 & 7 \\ 5 & 2 & 4 & 8 \end{bmatrix}$
- $D = \begin{bmatrix} -2 & 3 & 1 & 7 \\ 5 & 2 & 4 & 8 \\ 0 & 3 & 0 & -4 \\ 0 & 0 & -5 & 0 \end{bmatrix}$

Problem 2.15 Let A be a $n \times n$ square matrix, and let k be a constant. How are $\det(kA)$ and $\det(A)$ related?

Problem 2.16 Let A be a $n \times n$ square matrix. Let B be the matrix obtained when the first two columns of A are interchanged. How are $\det(A)$ and $\det(B)$ related?