

Math 4050

Practice Problem Set #12

At the top of your write-up, you must also write a statement attesting that you have at least thought about all assigned problems. Points will be deducted if you do not write this statement. This does not mean that you solved all of the problems — just that you gave some thought about how to solve every problem. For the sake of preparing for the state certification exam, as well as for your own integrity, I'd prefer that you are honest when writing this statement.

Problem 12.1 The height of a cannonball (in meters) is given by

$$s(t) = 100t - 5t^2,$$

where t is in seconds. A passing bird gets in the way of the cannonball 180 meters above the ground. Calculate the possible velocities of the cannonball at impact. Hint: $180 = 5 \times 36$.

Problem 12.2 Below the following descriptions, draw a sketch of a function f that satisfies the given properties. If the properties can't be satisfied, write "Impossible."

- f is continuous on $[0, 5]$
- f has an absolute maximum at the point $(1, 4)$
- f has an absolute minimum at the point $(4, 1)$

Problem 12.3 Below the following descriptions, draw a sketch of a function f that satisfies the given properties. If the properties can't be satisfied, write "Impossible."

- f is continuous on $[0, 5]$
- $f''(4) > 0$
- f has an absolute maximum at the point $(4, 5)$.

Problem 12.4 Below the following descriptions, draw a sketch of a function f that satisfies the given properties. If the properties can't be satisfied, write "Impossible."

- f has a critical point at $x = 2$
- f has a local minimum at $x = 2$
- $f'(2) \neq 0$

Problem 12.5 Below the following descriptions, draw a sketch of a function f that satisfies the given properties. If the properties can't be satisfied, write "Impossible."

- f is continuous
- f has a local minimum at $x = 2$
- $f'(x) > 0$ if $x < 2$
- $f'(x) < 0$ if $x > 2$

Problem 12.6 Draw a sketch of a function f that satisfies the given properties. If the properties can't be satisfied, write "Impossible."

- f has a absolute minimum at $x = 2$
- f has a critical point at $x = 2$
- $f'(2) \neq 0$

Problem 12.7 A watermelon is dropped from a height of 400 feet above the ground. How fast is it going when it hits the ground? *Hint:* Gravity pulls downward at a constant $g = 32 \text{ ft/s}^2$.

Problem 12.8 Draw a sketch of a function f that satisfies the given properties. If the properties can't be satisfied, write "Impossible."

- f is continuous and differentiable on the interval $[1, 5]$.
- The condition of the Mean Value Theorem is satisfied at $x = 2$:

$$f'(2) = \frac{f(5) - f(1)}{5 - 1}$$

Problem 12.9 Draw a sketch of a function f that satisfies the given properties. If the properties can't be satisfied, write "Impossible."

- f has a critical point at $x = 2$
- f has neither a local minimum nor a local maximum at $x = 2$

Problem 12.10 Let f and g be continuous functions so that $f(0) > g(0)$ and $f(1) < g(1)$.

- Draw a picture of f and g that corresponds with these assumptions.
- Use the Intermediate Value Theorem to prove that there is at least one point c in between 0 and 1 so that $f(c) = g(c)$.
- Draw this point c in your figure of part (a).

Problem 12.11 A ball is dropped from a bridge 80 feet high. At the same time that the first ball is dropped, directly below on the ground, a second ball is fired upward with an initial velocity of 40 feet per second. For both balls, the acceleration due to gravity is -32 ft/s^2 .

Determine how long it takes for the two balls to collide.