Math 4050 — Practice Problem Set #11

At the top of your write-up, you must also write a statement attesting that you have at least thought about all assigned problems. Points will be deducted if you do not write this statement. This does not mean that you solved all of the problems — just that you gave some thought about how to solve every problem. For the sake of preparing for the state certification exam, as well as for your own integrity, I'd prefer that you are honest when writing this statement.

Problem 11.1 Sketch the graph of

$$f(x) = \frac{x^2}{x^2-4} = \frac{x^2}{(x-2)(x+2)}$$

Be sure to label all intercepts, local extrema, and points of inflection. *Big Hint:* You may use the following information to help you get started:

$$\begin{aligned} f'(x) &= \frac{-8x}{(x^2-4)^2} = \frac{-8x}{(x-2)^2(x+2)^2} \\ f''(x) &= \frac{8(3x^2+4)}{(x^2-4)^3} = \frac{8(3x^2+4)}{(x-2)^3(x+2)^3} \end{aligned}$$

Problem 11.2

- Find the linearization of the function $f(x) = \sqrt{x^2+9}$ at $a = 4$.
- Use the linearization found in part (a) to estimate $f(4.1)$. Your answer should be accurate to three decimal places.

Problem 11.3 Find the absolute maximum and absolute minimum of

$$f(x) = \frac{\sin x}{2-\cos x}$$

on the interval $[0, \pi]$.

Problem 11.4 Kevin stands at a street corner. Britney stands 240 feet to the east of Kevin when they first see each other. At the same time, Britney runs west at a constant rate of 8 ft/s, while Kevin walks north at a constant rate of 3 ft/s. How fast is the angle θ, measuring the angle from Britney to the street corner and to Kevin, changing when Kevin has walked 60 feet?

Problem 11.5 Use Newton's method to estimate a root of $x^3 - 2x - 1 = 0$. Begin with $x_0 = 1$ and find x_2.

Problem 11.6 A rectangle lies in the first quadrant which has one side on the $x-$axis, one side on the $y-$axis, and a corner on the curve $y = \sqrt{6-x}$. Find the dimensions of the rectangle with maximum area.

Problem 11.7

- Suppose the cost of producing x computers is $C(x) = 4000 + 400x - 0.1x^2$. Find the marginal cost of producing 100 computers.
- In a sentence, explain what your answer to part (a) actually means.

Problem 11.8 Find the critical values of

$$f(x) = (x^2 - 4x)^{2/3}$$

Problem 11.9 Sketch the graph of $f(x) = x^4 + 4x^3$. Be sure to label all intercepts, local extrema, and points of inflection.

Problem 11.10 The electric field of an electric dipole along the axis of the dipole is

$$E(z) = kq\left(\frac{1}{(z-a)^2} - \frac{1}{(z+a)^2}\right),$$

where k is a constant, q is the charge, and z is the distance from the center of the dipole. If z is much larger than a, use linearization to show that

$$E(z) \approx \frac{4kaq}{z^3}.$$

Hint: To begin, notice that

$$(z-a)^{-2} = \left(z\left[1 - \frac{a}{z}\right]\right)^{-2} = z^{-2}\left(1 - \frac{a}{z}\right)^{-2}$$

and

$$(z+a)^{-2} = \left(z\left[1 + \frac{a}{z}\right]\right)^{-2} = z^{-2}\left(1 + \frac{a}{z}\right)^{-2}.$$

Problem 11.11 You are videotaping a NASCAR race from a stand 132 feet from the track, following a car that is moving at 180 mi/h (264 ft/s). How fact will your camera angle θ be changing a half-second after the car is right in front of you? Be sure to give the units of your answer.

Problem 11.12 The length of time (in hours) that a model airplane can spend in the air is given by

$$T(v) = \frac{10000v}{v^4 + 30000}, \qquad v \geq 0$$

where v is measured in miles per hour.

- Determine the value of v which maximizes $T(v)$.
- Find the maximum amount of time that the model airplane can be aloft.
- Verify that your solution is indeed the absolute maximum value of the function.