

Math 4050

Practice Problem Set #1

At the top of your write-up, you must also write a statement attesting that you have at least thought about all assigned problems. Points will be deducted if you do not write this statement. This does not mean that you solved all of the problems — just that you gave some thought about how to solve every problem. For the sake of preparing for the state certification exam, as well as for your own integrity, I'd prefer that you are honest when writing this statement.

Problem 1.1 Suppose that x is proportional to y and inversely proportional to the square of z . If $y = 10$ and $z = 5$, then $x = 2$. Find x if $y = 4$ and $z = 7$.

Problem 1.2 Let $f(x) = \frac{1}{x^2}$ and $g(x) = \sqrt{x - 16}$.

- Find $f \circ g$, and find the domain of the composition. Express the domain using interval notation.
- Find $g \circ f$, and find the domain of the composition. Express the domain using interval notation.

Problem 1.3 Let $f(x) = \frac{x + 1}{x - 2}$ and $g(x) = \frac{x + 4}{x - 3}$.

- Find the function fg , and find the domain of this function.
- Find the function f/g , and find the domain of this function.

Problem 1.4 A function f is given, and the indicated transformations are applied to its graph in the given order. Write and simplify the equation for the final transformed graph.

- $f(x) = x^2$
- Stretch horizontally by a factor of 3
- Shift to the left 2 units
- Reflect across the x -axis
- Stretch vertically by a factor of 4
- Shift upward 5 units

Problem 1.5 Find the inverse of the function

$$f(x) = \frac{3x + 1}{x - 2}.$$

Also, state the domain and the range of f^{-1} .

Problem 1.6 A man stands at a point A on the bank of a straight river which is 2 miles wide. To reach point B , 7 miles downstream from the opposite bank, he first rows his boat to a point P on the opposite bank and then walks the remaining distance x from P to B . He can row at a speed of 2 miles per hour and walk at a speed of 5 miles per hour.

Find a function that models the time needed for the trip in terms of the distance x .

Problem 1.7 Determine whether the function

$$f(x) = \frac{x^3 - x}{x^4 + 4x^2 + 1}$$

is even, odd, or neither.

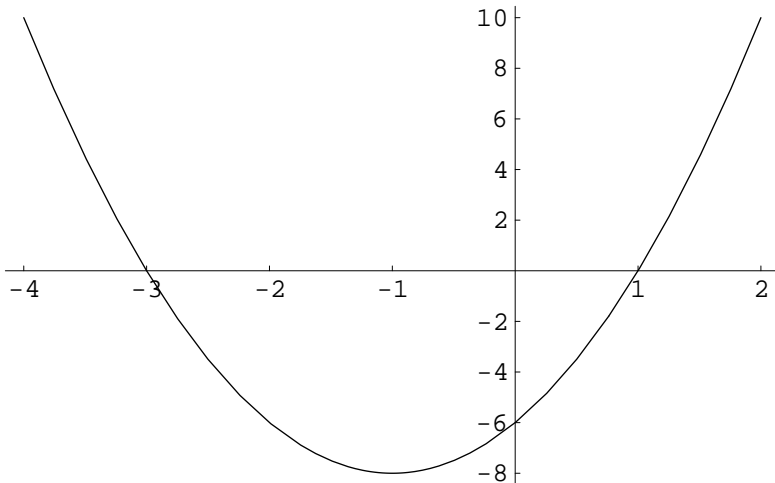
Problem 1.8 Sketch a function that does *not* have an inverse. (No computations are required for this problem.)

Problem 1.9 Sketch the graph of

$$f(x) = \begin{cases} x^2, & x \leq 1 \\ -2x + 5, & x > 1 \end{cases}$$

Problem 1.10 Find the equation of the quadratic function plotted below. You may write your answer in either the form

$$f(x) = ax^2 + bx + c \quad \text{or} \quad f(x) = a(x - h)^2 + k.$$



Problem 1.11

1. Find the set of all x so that $\sqrt{x^2} = x$. Express your answer in interval notation.
2. Find the set of all x so that $(\sqrt{x})^2 = x$. Express your answer in interval notation.

Problem 1.12 Estimate how many grains of sugar it would take to form a planet as big as the Earth. You will get full credit if your estimate is within a multiple of 10 of your instructor's estimate and you show logically correct work for producing your estimate. Express your answer in scientific notation with two significant digits (for example, 2.4×10^{24}).

Problem 1.13 Estimate the rate at which your hair is growing in miles per hour. You will get full credit if your estimate is within a multiple of 10 of your instructor's estimate and you show logically correct work for producing your estimate. Express your answer in scientific notation with two significant digits (for example, 2.4×10^{24}).

Problem 1.14 A plastic bag for holding produce at a grocery store has a width of 8 inches and a length of 15 inches. A child fills it with air to make an approximately spherical balloon. Estimate the volume of the balloon. You will get full credit if your estimate is within a multiple of 2 of your instructor's estimate and you show logically correct work for producing your estimate. Express your answer in scientific notation with two significant digits (for example, 2.4×10^{24}).

Problem 1.15 Estimate the number of people in the world who, at this very instant, are picking their noses. You will get full credit if your estimate is within a multiple of 10 of your instructor's estimate and you show logically correct work for producing your estimate. Express your answer in scientific notation with two significant digits (for example, 2.4×10^{24}).