Math 4050 **Practice Problem Set #9**

At the top of your write-up, you must also write a statement attesting that you have at least thought about all assigned problems. Points will be deducted if you do not write this statement. This does not mean that you solved all of the problems — just that you gave some thought about how to solve every problem. For the sake of preparing for the state certification exam, as well as for your own integrity, I'd prefer that you are honest when writing this statement.

Problem 9.1 Calculate

$$\lim_{x\to 0} \frac{\tan 2x}{5x}$$

Problem 9.2 Find a number δ so that

$$0 < |x-6| < \delta \Longrightarrow \left|\sqrt{16-2x} - 2\right| < \epsilon.$$

Problem 9.3 To what limit does the above $\delta - \epsilon$ argument correspond? Fill in the blanks:

$$\lim_{\square \to \square} \square = \square$$

Problem 9.4 If possible, sketch the graph of a function so that

- $f(0) = 0$, and
- $\lim_{x\to 0} f(x)$ is undefined.

If this cannot be done, write the word "Impossible."

Problem 9.5 If possible, sketch the graph of a function so that

- $\lim_{x\to 2^+} f(x) = 3$,
- $\lim_{x\to 2^-} f(x) = 3$,
- $f(x)$ is discontinuous at $x = 2$.

If this cannot be done, write the word "Impossible."

Problem 9.6 True or false, and explain: if $\lim_{x\to 2} g(x) = 6$, then $\lim_{x\to 2^+} g(x) = 6$ and $\lim_{x\to 2^-} g(x) = 6$.

Problem 9.7 True or false, and explain: $h(x) = \dfrac{x^2-1}{x-1}$ is continuous at $x = 1$.

Problem 9.8 Calculate

$$\lim_{x\to -\infty} \frac{x^2 - 5x + 4}{2x^2 + x - 10}$$

Problem 9.9 Calculate

$$\lim_{x\to 4^-} \frac{\sqrt{x} - 2}{x - 4}$$

Problem 9.10 Calculate

$$\lim_{x\to 2^-} \frac{x-3}{x^2+4x+4}.$$

Problem 9.11 Fill in the blank to make this sentence true, if possible: The function

$$f(x) = \begin{cases} \dfrac{x^2-x}{x^2-7x+6}, & x \neq 1, \\ \_\_\_\_\_, & x = 1 \end{cases}$$

is continuous at $x = 1$. If not possible, write the word "Impossible."

Problem 9.12 Calculate

$$\lim_{x\to 4^-} \frac{x^2-6x+8}{x^2-3x-4}$$

Problem 9.13 Fill in the blank to make this sentence true: The function

$$f(x) = \begin{cases} \dfrac{\sin 3x}{2x}, & x \neq 0, \\ \_\_\_\_\_, & x = 0 \end{cases}$$

is continuous at $x = 0$.