Math 3680 Review #3

Note: For all problems with small samples, you can assume that all hypotheses are satisfied so that the t-test can be used.

Review Problem 3.1 Define the following terms:

1. Null hypothesis

2. Alternative hypothesis

3. Critical value

4. Rejection region

5. Type I error

6. Type II error

7. Power

8. Significance level

9. Observed significance level

Review Problem 3.2 Suppose, after conducting a hypothesis test, the observed level of significance is determined to be $P = 0.50$. Which is the best conclusion?

1. H_0 is definitely false.

2. H_0 is definitely true.

3. There is a 50% chance that H_0 is true.

4. H_0 is plausible, and H_a is false.

5. H_0 is plausible

Review Problem 3.3 Eight vehicles are chosen at random from a fleet, and their emissions were measured under both highway driving and stop-and-go driving conditions. The results follow. Can we conclude that the mean level of emissions is less for highway driving than for stop-and-go driving?

	1	2	3	4	5	6	7	8
Stop-and-go	1500	870	1120	1250	3460	1110	1120	880
Highway	941	456	893	1060	3107	1339	1346	644

Review Problem 3.4 The rates that two machines can fill soft drink containers are compared. For a sample of 60 different minutes, Machine 1 filled an average of 73.8 cans per minute, with a standard deviation of 5.2 cans per minute. The similar statistics for Machine 2 are 76.1 and 4.1, respectively.

1. Is this enough information to think that Machine 2 is faster than Machine 1?

2. Determine the power of the test if Machine 2 can fill, on average, 3 cans per minute more than Machine 1.

3. Determine the number of cans that must be filled by both machines if it's desired that $\beta(3) = 0.001$.

4. Suppose that Machine 2 is significantly more expensive to test. Determine the number of machines that must be tested if it's desired that $\beta(3) = 0.001$ and Machine 1 should be tested twice as much as Machine 2.

Review Problem 3.5 Two extrusion machines that manufacture steel rods are being compared. In a sample of 1000 rods from Machine 1, 960 met specifications regarding length and diameter. In a sample of 600 rods from Machine 2, 582 met the specifications. Machine 2 is more expensive to run, and so it is decided that Machine 1 will be used unless it can be clearly shown that Machine 2 produces a larger proportion of rods meeting specifications.

1. Is this enough information to think that Machine 2 should be used?

2. Determine the probability of incorrectly deciding that the two machines have the same rate of meeting specifications if Machine 1 actually meets specifications 95% of the time while Machine 2 actually meets specifications 97.5% of the time.

3. Determine the number of times that machines must be equally tested if it's desired that the probabiilty in part 2 should be equal to 0.01.

Review Problem 3.6 Five measurements are taken of the octane rating of a high-grade gasoline: 90.1, 88.8, 89.5, 91.0, 92.1. Can you conclude that the mean octane rating is less than its advertised value of 91?

Review Problem 3.7 A shipment of fibers is acceptable if the mean breaking strength of the fibers of 50 N. On the other hand, the shipment is not acceptable if the mean breaking strength is less than 50 N. A sample of 80 fibers is tested from this shipment; the sample breaking strength is found to be 49.1 N with a standard deviation of 5.2 N.

1. Determine if the shipment is acceptable at the $\alpha = 0.01$ level of significance.

2. Find the power of the test if the mean breaking strength is actually 48 N.

3. Find the number of tests that are necessary if it's required that the probabiliy of making a Type II error should be 0.01 if the mean breaking strength is <u>48 N</u>.

Review Problem 3.8 A bank deems its credit-rating system to be satisfactory if it correctly assesses the worthiness of at least 80% of the bank's customers. In a random sample of 1000 clients, 740 were correctly assessed.

1. Is this enough evidence to think that the credit-rating system is satisfactory?

2. Find the power of the test if the credit-rating system is actually 75% accurate.

3. Determine the sample size necessary if it's desired that the power of the test be 0.995 if the credit-rating system is 75% accurate.

Review Problem 3.9 Good design can make Web navigation easier. A sample of 10 users using a conventional Web design averaged 32.3 items identified, with an SD of 8.56. A sample of 10 users using a new structured Web design averaged 44.1 items identified, with an SD of 10.09. Can we conclude that the mean number of items identified is greater with the new structured design?

Review Problem 3.10 State the assumption that must be checked before using a $t-$test for an average or else finding a confidence interval for an average based on a small sample size. Also, state how that assumption is actually checked.

Review Problem 3.11 A traffic study researcher was hired to ascertain the effect of wearing safety devices on driver reaction times. To investigate this question, he randomly selected 15 students from a driver education program. Each student performed two different simulated driving tasks — one wearing a safety device and one without. The reaction-time scores for both tests (in hundredth of seconds) are shown in the table below. Find a 90% confidence interval for the difference between reaction-time scores under these conditions.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Device	36.7	37.5	39.3	44.0	38.4	43.1	36.2	40.6	34.9	31.7	37.5	42.8	32.6	36.8	38.0
No device	36.1	35.8	38.4	41.7	38.3	42.6	33.6	40.9	32.5	30.7	37.4	40.2	33.1	33.6	37.5

Review Problem 3.12 Researchers compare the compression strength of boxes using two different methods. For the first method, the average strength of 10 boxes is found to be 807 pounds with a standard deviation of 27 pounds. For the second method, the average strength of 10 other (but identically produced) boxes is found to be 757 pounds with a standard deviation of 41 pounds. The researchers claim that "the difference... was found to be small compared to normal variation in compression strength between identical boxes." Do you agree? Use a 0.01 level of significance.

Review Problem 3.13 Marine biochemists study the biochemical properties of fast and slow muscles of crayfish. Twelve fast-muscle fiber bundles were extracted from one sample of crayfish and analyzed for uptake of the protein Ca^{2a}. The average amount was 0.57 moles/milligram, with a standard deviation of 0.104 moles/milligram. A similar analysis of twelve slow-muscle fiber bundles (from a second sample) yielded an average of 0.37 moles/milligram and a standard deviation of 0.035 moles/milligram. Find a 99% confidence interval for the difference between the protein uptake means of fast and slow muscles of crayfish.

Review Problem 3.14 A traffic engineer studies vehicular speeeds on a street which has changed the posted speed limit. When the posted speed limit was 30 mph, the engineer observed 49 vehicles violating the speed limit out of 100 randomly selected vehicles. When the posted speed limit was raised to 35 mph, the engineer observed 19 vehicles violating the speed limit out of 95 other randomly selected vehicles. Is there enough evidence to think that the rate of violations was higher for the 30 mph speed limit than for the 35 mph speed limit? Use a 0.01 level of significance.

Note: The above question is different than the version that was distributed in class.

Review Problem 3.15 To reduce costs, a bakery has implemented a new leavening process for preparing commercial bread loaves. Fifty loaves that were made using the old process were analyzed for calorie content, yielding an average of 1,330 calories and a standard deviation of 238 calories. For sixty loaves that were made using the new process, the average number of calories was 1,255 with a standard deviation of 215. Find a 99% upper confidence bound on the difference in the average calorie content for the two processes.

Note: The above question is different than the version that was distributed in class.

Review Problem 3.16 Some cities designate certain traffic lanes as only for carpooling. To evaluate the effectiveness of this plan, toll booth personnel monitored 2,000 randomly selected cars prior to designating carpooling lanes, and 652 of these were car-pool riders. In a study of 1,500 cars after they the designation, 576 were car-pool riders. Find a 99% confidence interval for the difference in carpooling rates from before and after the lane designation.

Review Problem 3.17 Wait staff at restaurants have employed various strategies to increase tips. One such strategy includes introducing themselves by name. In an experiment when a waitress introduced herself by name for 50 customers, the average tip amount was 19.45% of the bill, with a standard deviation of 6.23%. For 50 other customers when she did not introduce herself, the average tip amount was 13.34% of the bill, with a standard deviation of 5.10%. Does the data suggest that an introduction increases tips by more than 5 percentage points? Use a 0.01 level of significance.

Note: The above question is different than the version that was distributed in class.