1. State the definition of an infinite decimal (in base 10), and prove that an infinite decimal must converge to a real number.

2. Prove: The terminating decimal $D.d_1 \dots d_t$ (where $d_t > 0$) has a second decimal representation.

3. Convert the base-twelve duodecimal $3A.B5$ to a fraction, written in base 10.

4. Convert the base-sixteen hexadecimal $7.8\overline{AF}$ to a fraction, written in base 10.

5. Which of the following six assertions are true, and which are false? The lowest-terms fraction $\frac{m}{n}$ has a terminating decimal representation:

   - <u>if and only if</u> / <u>if</u> / <u>only if</u> n is divisible by no prime other than 2
   - <u>if and only if</u> / <u>if</u> / <u>only if</u> n is not divisible by 3

6. Prove that $\dfrac{11}{37} = 0.\overline{297}$. (It is not enough to simply plug into a calculator and see the repeating decimal, because you are not guaranteed that the pattern won't break after the digits shown on the calculator. Which problems below refer to this same issue?)

7. A rational number can be expressed as either a decimal, as a lowest-term fraction, and either in the form $\dfrac{M}{10^r}$ or $\dfrac{M}{10^r(10^l - 1)}$. Express each rational number below in the other two forms. Do as much work as you can without a calculator.

0.124	$0.\overline{386100}$	$0.0000\overline{27}$	$0.12\overline{345}$
$3/64$	$4/39$	$11/48$	$7/250$
$123/10000$	$3478/9999$	$49732/99999900$	$21497323/99999900$

8. Use the Division Algorithm and a calculator to find the decimal representation of 8/17. Repeat for 4567/2624.

9. Let $x = 4329218107/7792592592$. According to a calculator, $x = 0.555555556$. Does $x = 0.\overline{5}$?

10. Consider those reciprocals of primes that have simple-periodic decimal representations. Prove:

    - There is exactly one with period 1. What is it?
    - There is exactly one with period 2. What is it?
    - There is exactly one with period 3. What is it?
    - There is exactly one with period 4. What is it?

11. Consider those reciprocals of integers that have simple-periodic decimal representations. Prove:

    - There are exactly two with period 1. What are they?
    - There are exactly three with period 2. What are they?
    - There are exactly five with period 3. What are they?
    - There are exactly six with period 4. What are they?

12. Find a repeating, nonterminating decimal that begins as 0.222222222222222 (fifteen 2's) but is not equal to $0.\overline{2}$. Write your decimal as a fraction.

13. Prove that $x \in \mathbb{R} \setminus \mathbb{Q}$ if and only if x has an infinite nonrepeating decimal representation.

14. If $z \in \mathbb{C}$, prove that $|\text{Re}(z)| \le |z|$ and $|\text{Im}(z)| \le |z|$.
15. If $z \in \mathbb{C}$, prove that $|z| = |-z| = |\overline{z}|$.
16. If $z \in \mathbb{C}$, prove that $|z|^2 = z\overline{z}$.
17. If $z, w \in \mathbb{C}$, prove that $\overline{w+z} = \overline{w} + \overline{z}$.
18. If $z, w \in \mathbb{C}$, prove that $\overline{wz} = \overline{w} \cdot \overline{z}$.
19. If $a \in \mathbb{R}$ and $z \in \mathbb{C}$, prove that $\overline{az} = a\overline{z}$.
20. If $z, w \in \mathbb{C}$, prove that $|wz| = |w||z|$.
21. If $z \in \mathbb{C}$, prove that $\overline{\overline{z}} = z$.
22. If $z \in \mathbb{C}$, prove that $\text{Re}(z) = \dfrac{z+\overline{z}}{2}$ and $\text{Im}(z) = \dfrac{z-\overline{z}}{2i}$.
23. If $z, w \in \mathbb{C}$, prove that $|w+z| \le |w| + |z|$.
24. State the division algorithm for polynomials.
25. Give a counterexample that shows the division algorithm is false for $\mathbb{Z}[x]$.
26. State and prove the Remainder Theorem for polynomials.
27. State and prove the Factor Theorem for polynomials.
28. Prove: For any odd positive integer n, $x+c$ is a factor of $x^n + c^n$.
29. Explain when and why synthetic division works.