Math 4050 — Practice Problem Set #13

Problem 13.1 Find the average value of $y = \sin x$ on the interval $[0, \pi]$.

Problem 13.2 Solve the initial value problem

$$\begin{cases} \dfrac{dy}{dx} = \sec^2(2x) \\ y(\pi/6) = 1 \end{cases}$$

Problem 13.3 Evaluate

$$\int_{-1}^{4} |x|\, dx$$

Problem 13.4 Solve the initial value problem

$$\begin{cases} \dfrac{dy}{dx} = \sqrt{4x+1} \\ y(6) = 2 \end{cases}$$

Problem 13.5 Find the area enclosed by the graphs of $y = \sin x$ and $y = \sin 2x$ on the interval $[0, \pi]$. *Hint:* $\sin 2x = 2 \sin x \cos x$.

Problem 13.6 A triangular load is placed on the left half of a simple beam with constant length L. The deflection at the midpoint is given by

$$\delta = \frac{k}{L} \int_{0}^{L/2} (3L^2 - 4x^2)x^2\, dx.$$

Calculate δ. Your answer will involve k and L.

Problem 13.7 Use Simpson's Rule and $n = 4$ subintervals to estimate

$$\int_{0}^{1} \frac{4}{x^2+1}\, dx$$

to six decimal places. *Hint:* Your answer should look familiar.

Problem 13.8 Evaluate $\int x^2 \left[x + \cot(x^3)\csc(x^3)\right]\, dx$.

Problem 13.9 Evaluate $\int \left[\sin x + x^2\sqrt{x^3+1}\right]\, dx$.

Problem 13.10 Determine $\dfrac{dy}{dx}$ if $y = \int_{\pi/4}^{\sec x} \sin^2 t\, dt$.

Problem 13.11 Evaluate $\int_{-5/2}^{-3/2} x(2x+4)^5\, dx$.

Problem 13.12 Evaluate $\int (3 + \tan x \sec x)^2\, dx$.

Problem 13.13 A slightly tapered bar has circular cross sections and is 0.3 meters long. The left and right ends of the bar have diameters 0.02 and 0.05 meters, respectively. When torques are applied to both sides, the bar's angle of twist is

$$\phi = \int_0^{0.3} \frac{32T}{\pi G(0.02 + 0.1x)^4}\, dx,$$

where T and G are constants. Compute ϕ. Your answer will involve T and G.

Problem 13.14 Solve the initial value problem

$$\begin{cases} \dfrac{dy}{dx} = \sin^3\left(\dfrac{x}{2}\right)\cos\left(\dfrac{x}{2}\right) \\ \\ y\left(\dfrac{\pi}{2}\right) = 5 \end{cases}$$

Problem 13.15 Define

$$F(x) = \int_0^{\tan x} \frac{4}{t^2+1}\, dt.$$

- Fill in the blank with a number: $F(0) =$ ____________.
- Fill in the blank with a number: $F'(x) =$ ____________. (You will need to use a trigonometric identity to show that $F'(x)$ is a constant.)
- Solve for $F(x)$ in the initial-value problem in the previous problem.
- Compute $F(\pi/4)$. Then, in a couple of sentences, explain the connection between the above answer and your answer to the previous problem.