

Problem 12.1 The height of a cannonball (in meters) is given by

$$s(t) = 100t - 5t^2,$$

where t is in seconds. A passing bird gets in the way of the cannonball 180 meters above the ground. Calculate the possible velocities of the cannonball at impact. Hint: $180 = 5 \times 36$.

Problem 12.2 Below the following descriptions, draw a sketch of a function f that satisfies the given properties. If the properties can't be satisfied, write "Impossible."

- f is continuous on $[0, 5]$
- f has an absolute maximum at the point $(1, 4)$
- f has an absolute minimum at the point $(4, 1)$

Problem 12.3 Below the following descriptions, draw a sketch of a function f that satisfies the given properties. If the properties can't be satisfied, write "Impossible."

- f is continuous on $[0, 5]$
- $f''(4) > 0$
- f has an absolute maximum at the point $(4, 5)$.

Problem 12.4 Below the following descriptions, draw a sketch of a function f that satisfies the given properties. If the properties can't be satisfied, write "Impossible."

- f has a critical point at $x = 2$
- f has a local minimum at $x = 2$
- $f'(2) \neq 0$

Problem 12.5 Below the following descriptions, draw a sketch of a function f that satisfies the given properties. If the properties can't be satisfied, write "Impossible."

- f has a local minimum at $x = 2$
- $f'(x) > 0$ if $x < 2$
- $f'(x) < 0$ if $x > 2$

Problem 12.6 Draw a sketch of a function f that satisfies the given properties. If the properties can't be satisfied, write "Impossible."

- f has a absolute minimum at $x = 2$
- f has a critical point at $x = 2$
- $f'(2) \neq 0$

Problem 12.7 A watermelon is dropped from a height of 400 feet above the ground. How fast is it going when it hits the ground? *Hint:* Gravity pulls downward at a constant $g = 32 \text{ ft/s}^2$.

Problem 12.8 Draw a sketch of a function f that satisfies the given properties. If the properties can't be satisfied, write "Impossible."

- f is continuous and differentiable on the interval $[1, 5]$.
- The condition of the Mean Value Theorem is satisfied at $x = 2$:

$$f'(2) = \frac{f(5) - f(1)}{5 - 1}$$

Problem 12.9 Draw a sketch of a function f that satisfies the given properties. If the properties can't be satisfied, write "Impossible."

- f has a critical point at $x = 2$
- f has neither a local minimum nor a local maximum at $x = 2$

Problem 12.10 Let f and g be continuous functions so that $f(0) > g(0)$ and $f(1) < g(1)$.

- Draw a picture of f and g that corresponds with these assumptions.
- Use the Intermediate Value Theorem to prove that there is at least one point c in between 0 and 1 so that $f(c) = g(c)$.
- Draw this point c in your figure of part (a).

Problem 12.11 A ball is dropped from a bridge 80 feet high. At the same time that the first ball is dropped, directly below on the ground, a second ball is fired upward with an initial velocity of 40 feet per second. For both balls, the acceleration due to gravity is -32 ft/s^2 .

Determine how long it takes for the two balls to collide.