Math 4050 **Practice Problem Set #8**

Problem 8.1 Find the coefficient of the x^9-term in the expansion of $(2x^3-1)^{12}$. *You do not need to give the entire expansion to receive full credit.*

Problem 8.2 Compute $\displaystyle\sum_{k=1}^{\infty} 2\left(-\frac{2}{5}\right)^{2k-1}$.

Problem 8.3 Express in $\sum-$notation:

$$q^2 - pq^6 + \frac{p^2q^{10}}{2} - \frac{p^3q^{14}}{6} + \frac{p^4q^{18}}{24} - \frac{p^5q^{22}}{120}$$

Problem 8.4 Use mathematical induction to show that

$$2 + 2^2 + 2^3 + \ldots + 2^n = 2^{n+1} - 2$$

Problem 8.5 For an arithmetic sequence, $a_4 = 35$ and $a_8 = 75$. Find the sum of the first 100 terms.

Problem 8.6 Consider the sequence recursively defined by

$$a_n = \begin{cases} 1, & n = 1 \\ a_{n-1} + 1 + 2\sqrt{a_{n-1}}, & n \geq 2 \end{cases}$$

- Find a_1, a_2, a_3, and a_4.
- Now guess a formula for a_n, and use mathematical induction to prove that your formula works.

Problem 8.7 Expand and simplify $(1-\sqrt{2})^6$.

Problem 8.8 Use the formula

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}.$$

to calculate

$$\sum_{k=1}^{12}\left(\frac{k^2}{5} + 2\right)$$

Problem 8.9 Evaluate the arithmethic series

$$0.9 + 1.3 + 1.7 + \ldots + 8.1 + 8.5$$

Problem 8.10 Use mathematical induction to show that $2^{2n-1} + 1$ is always divisible by 3.

Problem 8.11 Use mathematical induction to show that

$$1 + 3 + 5 + \ldots + (2n-1) = n^2$$

Problem 8.12 Compute

$$\sum_{k=3}^{100}(2k-3)$$

Problem 8.13 An infinite geometric sequence has -1000 and 8 as its second and fifth terms, respectively. Find the sum of this infinite geometric series.

Problem 8.14 Use the principle of telescoping series to exactly evaluate

$$\sum_{n=1}^{99} \left([n+1]^4 - n^4 \right)$$

Problem 8.15 Use mathematical induction to show that

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}.$$